NUS | Computing
National University of Singapore

CS4248: Natural Language Processing

Lecture 7 — Sequences

Announcements

- Assignment 2
  - Goal: practice manual feature engineering
  - Only certain technologies already covered are allowed
    (to keep it fair + manual features are easier to explain/interpret)
- Midterm Survey
  - 1% of your course mark with only 5 min of your time

Deadline for all:
Mar 14, 11.59 pm

- Project
  - TEAMMATES: intra-project formative feedback
  - ungraded but monitored

Outline

- **Overview: Sequence Tasks**

- POS Tagging
  - What are Parts of Speech?
  - Why is this task important and challenging?

- Hidden Markov Models (HMM)
  - Basic setup and components
  - Core HMM tasks
    - Model Learning
    - Likelihood computation
    - Viterbi decoding

Motivation

- So far: Bag-of-Words (BoW) models
  - Bag = whole document (e.g., Naive Bayes, Vector Space Model)
  - Bag = context of a word (e.g, PPMI and Word2Vec embeddings)

- Natural language: word order matters! (can vary greatly between languages, though)

Bob kills mosquitoes using the book of Hamlet

vs.

Hamlet kills Bob using the book of mosquitoes

The food tastes good and does not look bad

vs.

The food tastes bad and does not look good

Same words, very different meanings!

Motivation — Example: English

- Fundamental rules word order
  - Subject—Verb—Object (SVO)
  - Adjectives only (immediately) before nouns
  - …and many more
- "Informal" rules — e.g.: order of adjectives
  - Rule: opinion→size→physical quality→shape→age→color→origin→material→type→purpose

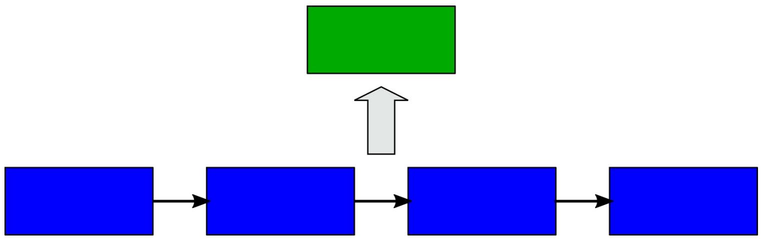
I saw a beautiful, old, blue, German car.

vs.

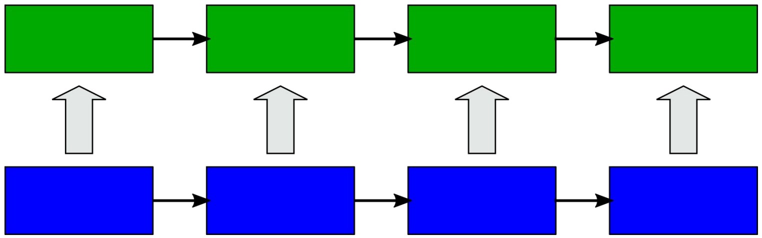
I saw a German, blue, beautiful, old car.

Types of Sequence Tasks

- Sequence classification
  (Many-to-One, N→1)
  - Sentiment analysis
  - Document categorization

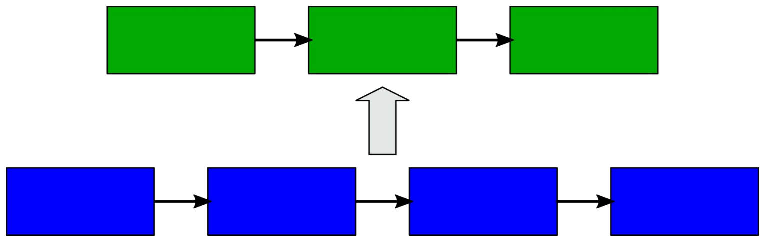


- Sequence labeling/tagging
  (Many-to-Many, N→N)
  - Part-of-Speech Tagging
  - Named Entity Recognition

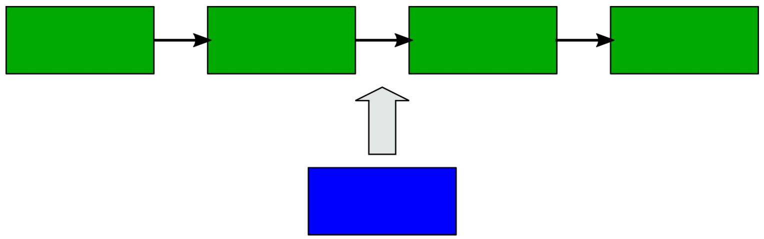


Types of Sequence Tasks

- Sequence translation
  (Many-to-Many, N→M)
  - Machine translation
  - Sentence simplification
  - Text summarization



- Sequence generation
  (One-to-Many, 1→N)
  - Image captioning



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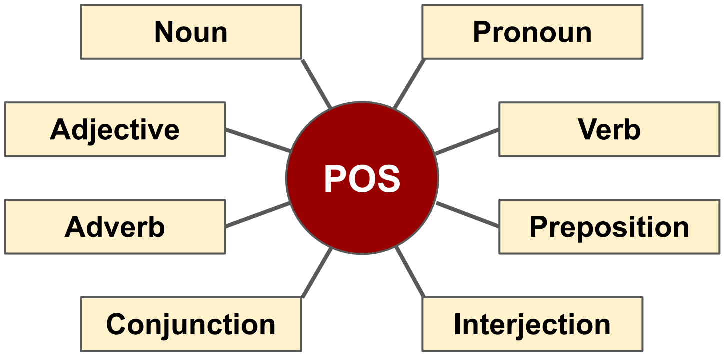
Part-of-Speech Tagging

- Part of Speech (also: word class or syntactic category)
  - Each word belongs one or more of these classes
  - English: 8 main parts of speech / word classes
    (many additional classes and subclasses considered in practice)



- Part-of-Speech (POS) tagging
  - Assign each word in a text a part of speech (duh!)

proper noun singular	verb past tense	adverb	preposition or subordinating conjunction		possessive pronoun	adjective	noun, singular or mass	punctuation
NNP	**VBD**	**RB**	**IN**	**IN**	**PRP$**	**JJ**	**NN**	**.**
Bob	*walked*	*slowly*	*because*	*of*	*his*	*swollen*	*ankle*	*.*

Penn Treebank Tag-Set

Base set: 36 POS tags

CC	Coordinating conjunction	*and, or*
CD	Cardinal number	*1, 2, 3, one, two, three*
DT	Determiner	*the, a, an, any, some*
EX	Existential there	
FW	Foreign word	
IN	Preposition / subord. conjunction	*in, into, whether, if*
JJ	Adjective	*cleaner, nice*
JJR	Adjective (comparative)	*cleaner, nicer*
JJS	Adjective (superlative)	*cleanes, nicest*
LS	List item marker	
MD	Modal	*can, could, may*
NN	Noun (singular or mass)	*machine, computer, air*
NNS	Noun (plural)	*machines, computers*
NNP	Proper noun (singular)	*Clementi Mall*
NNPS	Proper noun (plural)	*Americas*
PDT	Predeterminer	*all, both, half*
POS	Possessive ending	*'s*
PRP	Personal pronoun	*him. himself, we*

PP$	Possessive pronoun	*her, our, ours*
RB	Adverb	*quickly, swiftly*
RBR	Adverb (comparative)	*further, greater, more*
RBS	Adverb (superlative)	*furthest, greatest, most*
RP	Particle	*across, up*
SYM	Symbol	*=, +, &*
TO	to	*to*
UH	Interjection	*shucks, heck, oops*
VB	Verb (base form)	*be, assign, run*
VBD	Verb (past tense)	*was, assigned, ran*
VBG	Verb (gerund / present participle)	*being, assigning*
VBN	Verb (past participle)	*been, assigned*
VBP	Verb (non-3rd pers. sing. present)	*am, are*
VBZ	Verb (3rd pers. sing. present)	*is*
WDT	wh-determiner	*that, which, what*
WP	wh-pronoun	*that, which, whom*
WP$	Possessive wh-pronoun	*whose*
WRB	wh-adverb	*how, however, why*

Penn Treebank Tag-Set

Extended set: 12 tags for punctuations and special symbols

#	Pound sign	#
$	Dollar sign	$
.	Sentence-final punctuation	. ? !
:	Sentence-middle punctuation	: ; … - —
,	Comma	,
(	Left bracket character	([{ <
)	Right bracket character	)] } >
"	Straight double quote	
`	Left open single quote	
``	Left open double quote	
'	Right close single quote	
"	Right close double quote	

Part of Speech — Two Broad Categories

- **Closed class** words
  - Small, fixed membership — reasonably easy to enumerate
  - Generally, short function words that “structure” sentences
  - Examples: prepositions, pronouns, participles, determiners, conjunctions, etc.

- **Open class** words
  - Impossible to completely enumerate
  - New words continuously being invented, borrowed, etc.
  - For most languages: nouns, verbs, adjectives, adverbs

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POS Tagging — Why is it Important?

- Very useful or crucial for many NLP downstream tasks
  - Named Entity Recognition (typically comprised of nouns and proper nouns)
  - Information extraction (e.g., verbs indicate relations between entities)
  - Parsing (information of word classes useful before creating parse trees)
  - Speech Synthesis/Recognition (e.g., noun "DIScount" vs. verb "disCOUNT")
  - Authorship Attribution (e.g., relative frequencies of nouns, verbs, adjectives, etc.)
  - Machine Translation (e.g., reordering of adjectives and nouns)

➜ POS tagging: important low-level NLP task

Quick Quiz

Which POS have the highlighted words in the following headlines?

"Teacher ***Strikes*** *Idle Kids"*

"Hershey ***Bars*** *Protest"*

A verb / noun

B noun / noun

C verb / verb

D noun / verb

POS Tagging — Why is it Difficult? And How Difficult?

- Our common problem: **Ambiguity**
  - Many common words have multiple meanings ➜ multiple POS

"light"

noun or **verb** or **adjective**?

"hit"

noun or **verb** (which tense)?

"apple"

noun or **proper noun**?

  - Often ambiguous even with additional context

    (even humans can often no agree on the correct labeling!)

"Flying planes can be dangerous"

verb or **adjective**?

"Fruit flies like a banana"

verb or **noun**?

verb or **preposition**?

POS Tagging — Why is it Difficult? And How Difficult?

- POS tagging in English
  - ~85% of word types are unambiguous (e.g., "quickly" is always an adverb, "Alice" is always a proper noun)
  - ~15% of word types are ambiguous — but those are quite common!

➜ 55-65% of word tokens are ambiguous
  - Ambiguous = 2 or more possible POS tags
  - Results depend on text corpus

Quick Quiz

Which word can be assigned the **most** POS tags?

A *light*

B *up*

C *to*

D *bank*

POS Tagging — Baseline Algorithm

- Most straightforward approach
  - Label each word with its most frequent POS tag
  - Label unknown words as nouns (most common open world class)

➜ Result: ~92% accuracy (vs. ~97-98% accuracy for SOTA methods)
  - Doesn't sound so bad right?
  - 2 main problems:

    (1) **Imbalanced errors**
      - High accuracy due to common/frequent unambiguous words (e.g., "the", "a/an", "and", "or")
      - Many of these words also often not that interesting for downstream NLP tasks

    (2) **Downstream error propagation**
      - POS tagging as low-level NLP task ➜ errors quickly propagate up

POS Tagging — Unsupervised Algorithms

- Basic intuition
  - Utilize words with unambiguous POS tags ➜ **anchor words**
  - Observe patterns to group words into clusters of the same word class
  - Use anchor words to assign clusters (and each containing word) to a POS tag
- Practical considerations
  - No need for hand-labeled text corpora (only lexicon of anchor words required)
  - Poorer performance compared to supervised methods

POS Tagging — Supervised Methods

- Require hand-labeled text corpus
  - Used as input training data for supervised models
  - Challenging for low-resource languages
    (i.e., languages lacking in large, annotated datasets)

- Popular models (all yielding quite similar SOTA results)
  - Hidden Markov Models (HMM)
  - Conditional Random Fields (CRF)
  - Neural sequence models (RNNs, Transformers)
  - Large language models (e.g., BERT)

- Accuracies have reached "human ceiling"
  (i.e., POS taggers as good as human annotators)
- POS tagging considered a solved task for high-resource languages (e.g. English)
- Limitations: low-resource languages and special application domains

Quick Quiz

Which word is a **anchor word**?

(i.e., which word has only 1 POS?)

A *sun*

B *venus*

C *earth*

D *mars*

In-Lecture Activity (5 min)

- Question: What are other examples of words with many POS?
  - The more the better, but at least 3 different POS
  - Post your solution to the Canvas Discussion

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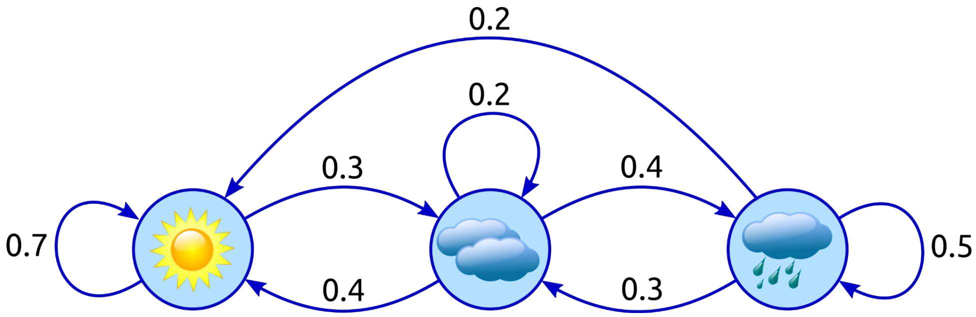
Markov Chains

- Markov Chain
  - Models transitions between a set of states using transition probabilities (captured by a transition matrix A)
  - Transition only depends on current state (Markov assumption)
  - Sequence: series of transitions

- Example: "Daily Weather"
  - 3 states: *sunny*, *cloudy*, *rainy*



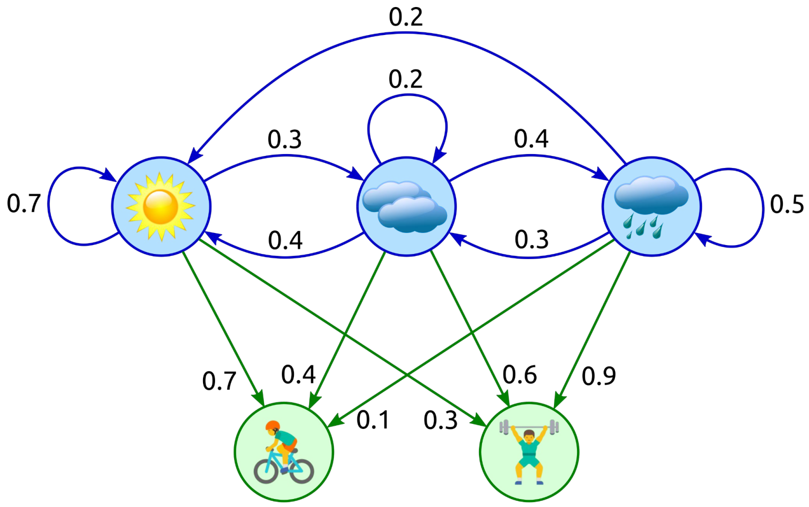
$$A = \begin{bmatrix} 0.7 & 0.3 & 0.0 \\ 0.4 & 0.2 & 0.4 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

Example question: *"What is the probability of getting a 5 sunny days in a row?'*

Markov Chain ➜ Hidden Markov Models

- Hidden Markov Models (HMM)
  - States are hidden (i.e., not directly observable)
  - Observable variables that depend on the states

- Example: "Exercising Routine"
  - 3 hidden(!) states: *sunny*, *cloudy*, rainy
  - 2 observed activities: *biking*, *lifting*
    (with the activity depending on the weather)



Example question: *"Given that Chris went first 3 days lifting and then 3 days biking, what was the most likely weather over the last 6 days?"*

HMM — Components

Finite Set of **states** $S = \{s_1, s_2, ..., s_N\}$

Sequence of states

$$Q = q_1, q_2, q_3, \ldots, q_T \text{ , with } q_t \in S$$

Finite set of **symbols** $V = \{v_1, v_2, ..., v_M\}$

Sequence of observations

$$O = o_1, o_2, o_3, \ldots, o_T \text{ , with } o_t \in V$$

Transition probability matrix A

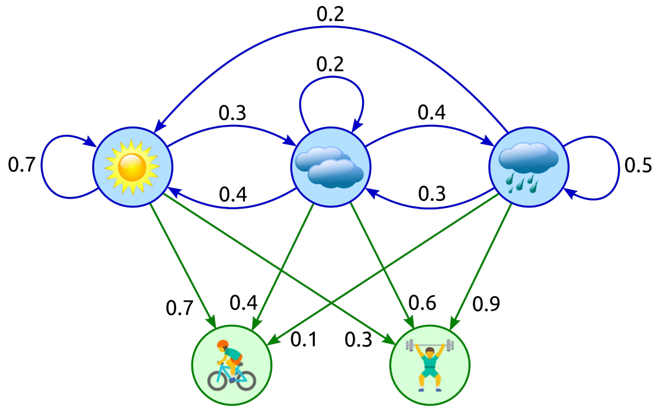
$$A = \{a_{ij}\}, \quad a_{ij} = P(q_{t+1} = s_j |\ q_t = s_i)$$

Observation / emission probability matrix B

$$B = \{b_i(o_k)\}, \quad b_i(o_k) = P(o_t = v_k |\ q_t = s_i)$$

Initial state distribution π

$$\pi = \{\pi_i\}, \quad \pi_i = P(q_1 \mid s_i)$$



$$A = \{a_{ij}\} = \begin{array}{c} \\ ☀️ \\ ☁️ \\ 🌧️ \end{array} \begin{array}{c} \begin{array}{ccc} ☀️ & ☁️ & 🌧️ \end{array} \\ \begin{bmatrix} 0.7 & 0.3 & 0.0 \\ 0.4 & 0.2 & 0.4 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

$$B = \{b_i(o_k)\} = \begin{array}{c} \\ ☀️ \\ ☁️ \\ 🌧️ \end{array} \begin{array}{c} \begin{array}{cc} 🚴 & 🏋️ \end{array} \\ \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \\ 0.1 & 0.9 \end{bmatrix} \end{array}$$

HMM — Probabilities (annotated)

Transition probability matrix A

$$A = \{a_{ij}\}, \quad a_{ij} = P(q_{t+1} = s_j \mid q_t = s_i)$$

Probability of transitioning from state s_i to s_j at any time t

$$\sum_j^N a_{ij} = 1 \quad \forall i$$

Observation / emission probability matrix B

$$B = \{b_i(o_k)\}, \quad b_i(o_k) = P(o_t = v_k \mid q_t = s_i)$$

Probability of state s_i generating output v_k at any time t

$$\sum_k^M b_i(o_k) = 1, \quad \forall k$$

Initial state distribution π

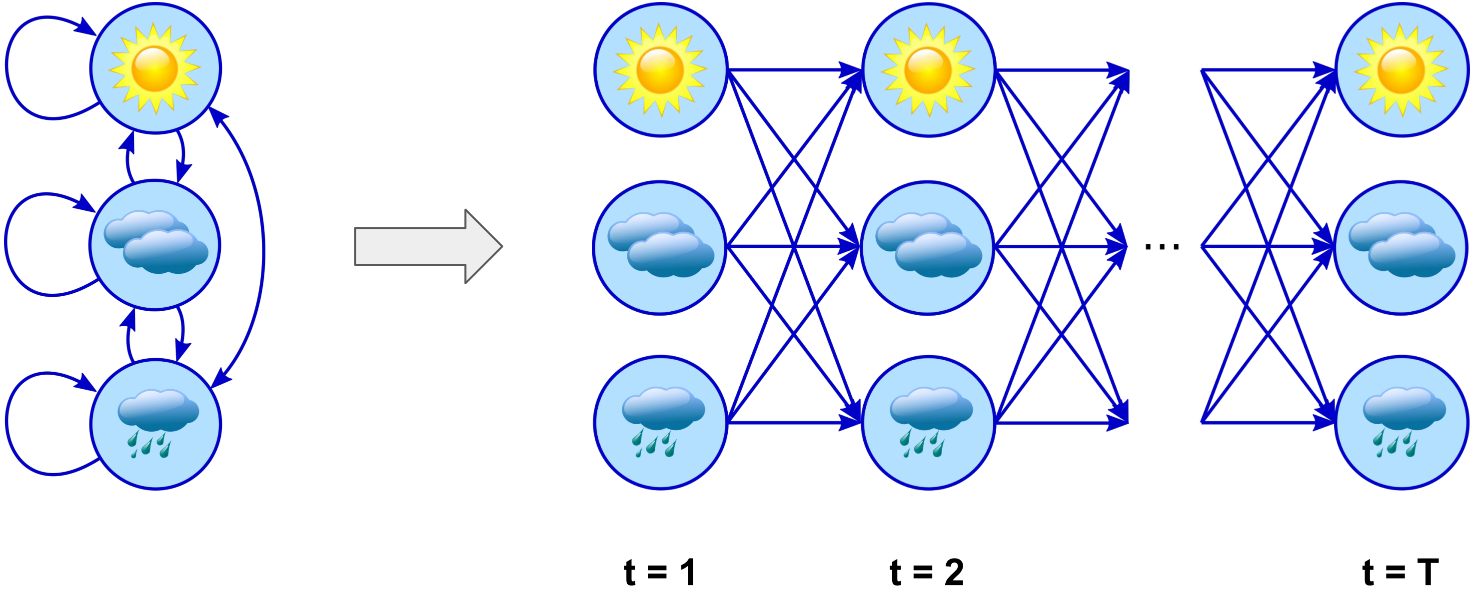
$$\pi = \{\pi_i\}, \quad \pi_i = P(q_1 = s_i)$$

Probability of sequence starting in state s_i

$$\sum_i^N \pi_i = 1$$

HMM — Unrolled Representation

Trellis diagram — graph representation of all possible states and transitions over time



Quick Quiz

Is the Hidden Markov Model a **generative** model or a **discriminative** model?

A Generative

B Discriminative

C Both

D Neither

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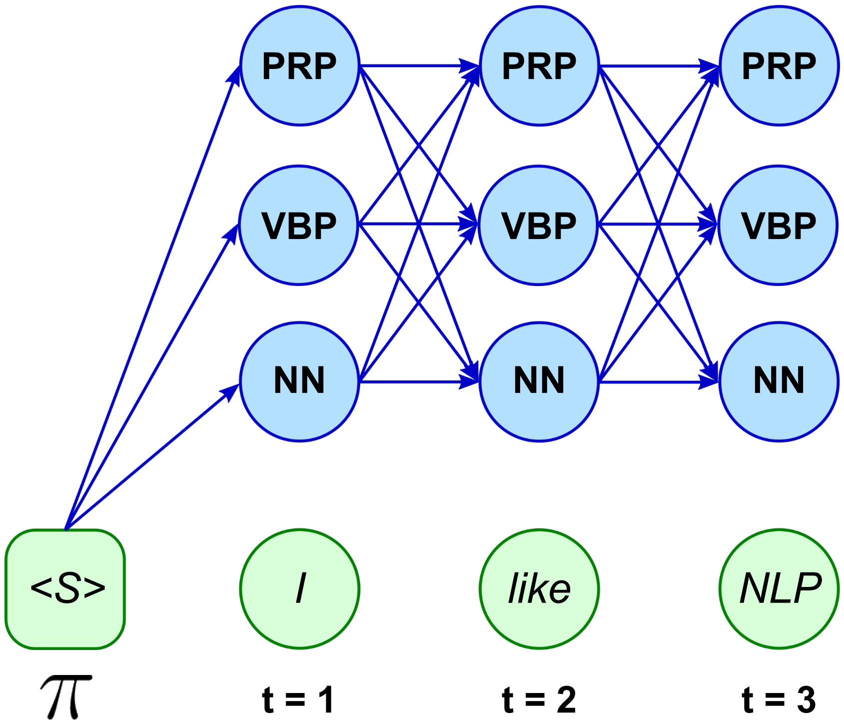
POS Tagging with HMMs

- Basic setup
  - Hidden states ➜ POS tags
  - Observations ➜ words
- Example
  - 3 states: {PRP, VBP, NN}



HMM — Core Tasks

(1) Model Learning

Given corresponding state and observation sequences Q and O
➜ Learn all model parameters, i.e., probabilities A, B and π

Training using an annotated dataset

(2) Likelihood

Given an HMM $\theta = (A, B, \pi)$
+ an state sequence Q
+ an observation sequence O
➜ Calculate the probability $P(Q, O|\theta)$

Given 2 POS tag sequences for a sentence, compare which is more likely

(3) Decoding

Given an HMM $\theta = (A, B, \pi)$ + an observation sequence O
➜ Find the most likely sequence of states

Given a sentence, find the most likely POS tags

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HMM — Model Learning

Quick Quiz: Spotting a familiar issue? How can we address it?

- Calculating probabilities using Maximum Likelihood Estimates

$$\pi_i = P(q_1 = s_i) = \frac{Count(\langle S \rangle s_i)}{Count(\langle S \rangle)}$$

#sentences starting with state s_i

#sentences

$$a_{ij} = P(q_{t+1} = s_j | \, q_t = s_i) = \frac{Count(s_i s_j)}{Count(s_i)}$$

#occurences of state s_i followed by state s_j

#occurences of state s_i

$$b_i(o_k) = P(o_t = v_k | \, q_t = s_i) = \frac{Count(v_k, s_i)}{Count(s_i)}$$

#occurences of observation v_k in state s_i

#occurences of state s_i

HMM — Model Learning — Side Note

- POS tagging using HMM
  - Full supervised task ➜ corpus of words labeled with all the correct POS tags
  - Fully "visible" Markov Model
    (we have the state and observation sequences)

"Direct" parameter learning using MLE
(we just need simple counts)

- Often in other applications
  - State sequences Q are not known
  - Impossible to compute simple counts

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Likelihood

- Given: HMM $\theta = (A, B, \pi)$ and

$$O = o_1, o_2, o_3, \ldots, o_T$$

$$Q = q_1, q_2, q_3, \ldots, q_T$$

- Calculate joint probability $P(O, Q|\theta)$

$$P(O, Q|\theta) = P(O|Q) \cdot P(Q) = \prod_{i=1}^{T} \underbrace{P(o_i|q_i)}_{\text{emission probabilities}} \cdot \underbrace{P(q_i|q_{i-1})}_{\text{transition probabilities}}$$

with $\underbrace{P(q_1|q_0)}_{\text{initial state probability}} = P(q_1)$

Likelihood — Example

$$P(O,Q|\theta) = P(O|Q) \cdot P(Q) = \prod_{i=1}^{T} P(o_i|q_i) \cdot P(q_i|q_{i-1})$$

- Given: sentence O, POS tags Q

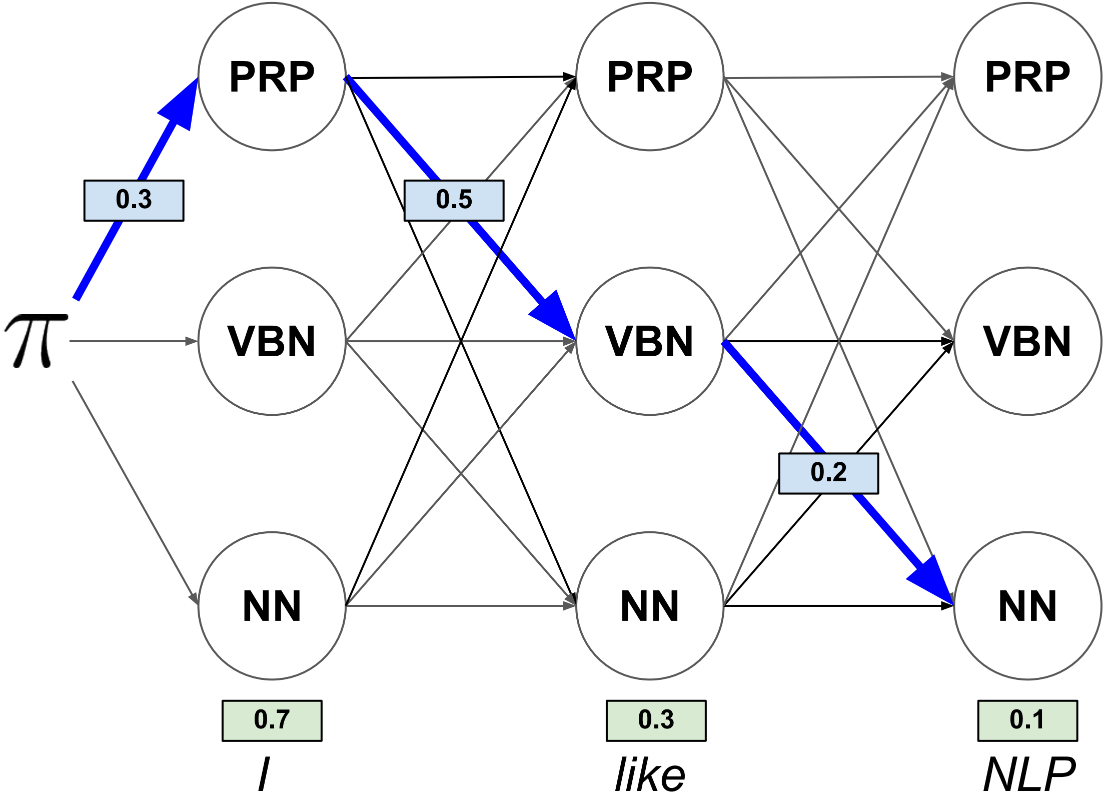
$$\left.\begin{array}{l} O = I, like, NLP \\ \\ Q = PRP, VBN, NN \end{array}\right\} \quad P(\text{"}I, like, NLP\text{"}|PRP\text{–}VBN\text{–}NN) = ?$$

$$\begin{aligned} P(\text{"}I, like, NLP\text{"}|PRP - VBN - NN) = \; & \underbrace{\begin{aligned} & P(I|PRP) \cdot P(PRP|\langle S \rangle) \cdot \\ & P(like|VBN) \cdot P(VBN|PRP) \cdot \\ & P(NLP|NN) \cdot P(NN|VBN) \end{aligned}} \end{aligned}$$

All values can be directly taken from A, B, and π

Likelihood — Example

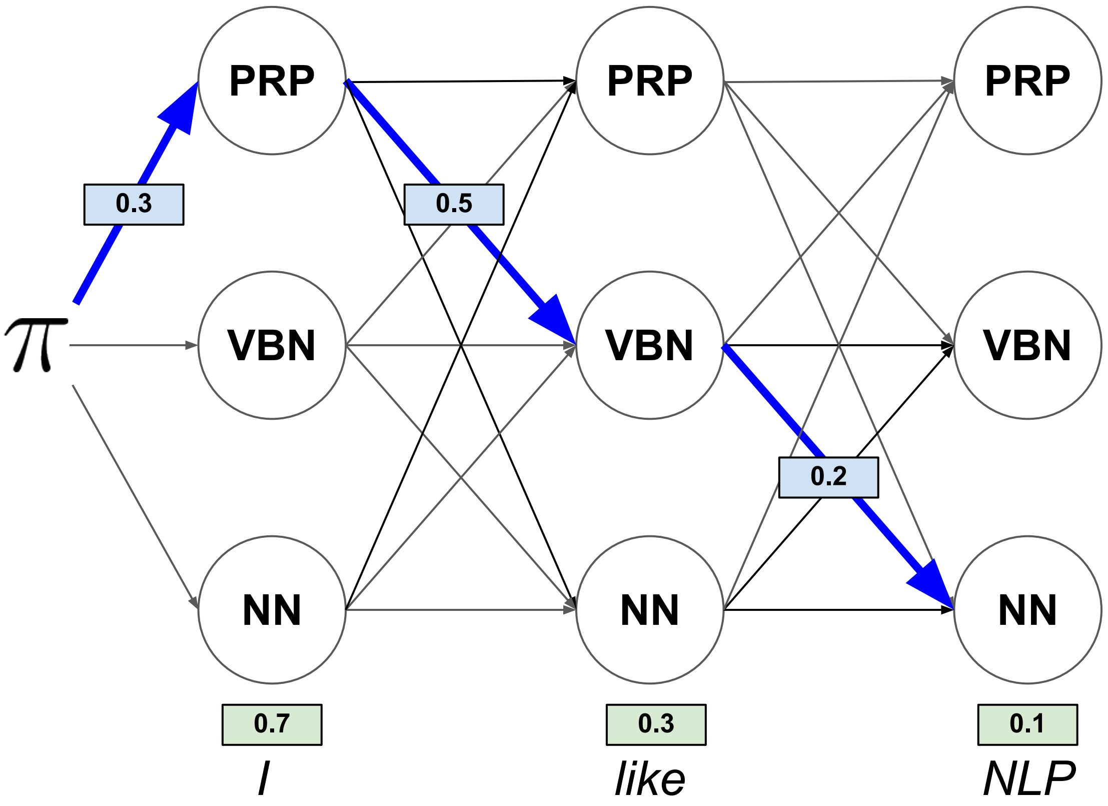
Visualization using a Trellis diagram ➜ just follow the path



$$P(\text{"}I, like, NLP\text{"}|PRP - VBN - NN) =$$
$$P(I|PRP) \cdot P(PRP|\langle S\rangle)\cdot$$
$$P(like|VBN) \cdot P(VBN|PRP)\cdot$$
$$P(NLP|NN) \cdot P(NN|VBN)$$

Likelihood — Example

Visualization using a Trellis diagram ➜ just follow the path



$$P("I, like, NLP"|PRP - VBN - NN) =$$
$$P(I|PRP) \cdot P(PRP|\langle S \rangle) \cdot$$
$$P(like|VBN) \cdot P(VBN|PRP) \cdot$$
$$P(NLP|NN) \cdot P(NN|VBN)$$

$$P("I, like, NLP"|PRP - VBN - NN) =$$
$$0.7 \cdot 0.3 \cdot$$
$$0.3 \cdot 0.5 \cdot$$
$$0.1 \cdot 0.2$$
$$= 0.00063$$

Likelihood — Can we decode with it?

- Naive algorithm for decoding (for a given observation sequence O)
  - Enumerate all possible state sequences Q
  - Compute all joint probabilities $P(O, Q)$
  - Return state sequence Q with highest joint probability

➜ What is the **runtime** of this algorithm?

Quick Quiz

What is the **runtime complexity** of this naive decoding algorithm?

(N = number of states; T = number of steps)

A $O(N^T \cdot T)$

B $O(N \cdot T)$

C $O(N^3 \cdot T)$

D $O(N^2 \cdot T^2)$

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Decoding

- Decoding task
  - Given an HMM $\theta = (A, B, \pi)$ + an observation sequence O
  - Find the most likely sequence of states Q

$$Q = \underset{q_1 \ldots q_t}{\operatorname{argmax}} \prod_{i=1}^{T} P(o_i|q_i) \cdot P(q_i|q_{i-1}) \qquad \text{with } P(q_1|q_0) = P(q_1)$$

emission probabilities

transition probabilities

initial state probability

➜ **Dynamic Programming** to avoid checking all possible state sequences

Viterbi Algorithm — Toy Example

- Oversimplified setup
  - 3 POS tags: **DT** (determiner), **NN** (noun), **VB** (verb)
  - Let's assume the following HMM

Note: The rows in B do not up to 1 since B does not capture all words, only those we needs.

$$\pi = \begin{array}{c} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

$$A = \begin{array}{cc} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} & \\ \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$

$$B = \begin{array}{cc} \begin{array}{cccc} \textit{the} & \textit{fans} & \textit{love} & \textit{show} \end{array} & \\ \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.0 & 0.05 & 0.3 & 0.1 \\ 0 & 0.25 & 0.15 & 0.3 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$

- Task: Find the most likely sequence of state (i.e., POS tags) for:

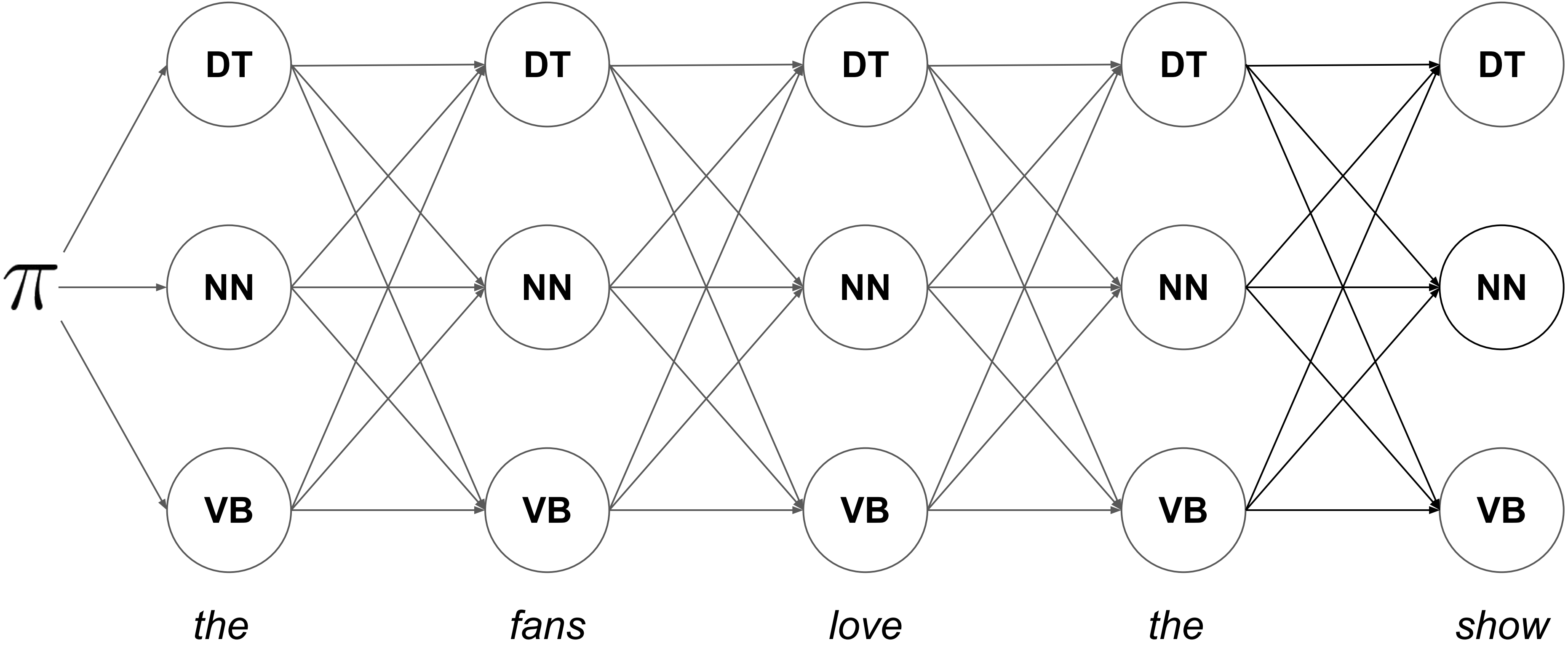
"the fans love the show"

Example

$$\pi = \begin{array}{c} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

$$A = \begin{array}{cc} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} & \\ \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$

$$B = \begin{array}{cc} \begin{array}{cccc} \textit{the} & \textit{fans} & \textit{love} & \textit{show} \end{array} & \\ \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.0 & 0.05 & 0.3 & 0.1 \\ 0 & 0.25 & 0.15 & 0.3 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$



Example

$$\pi = \begin{array}{c} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

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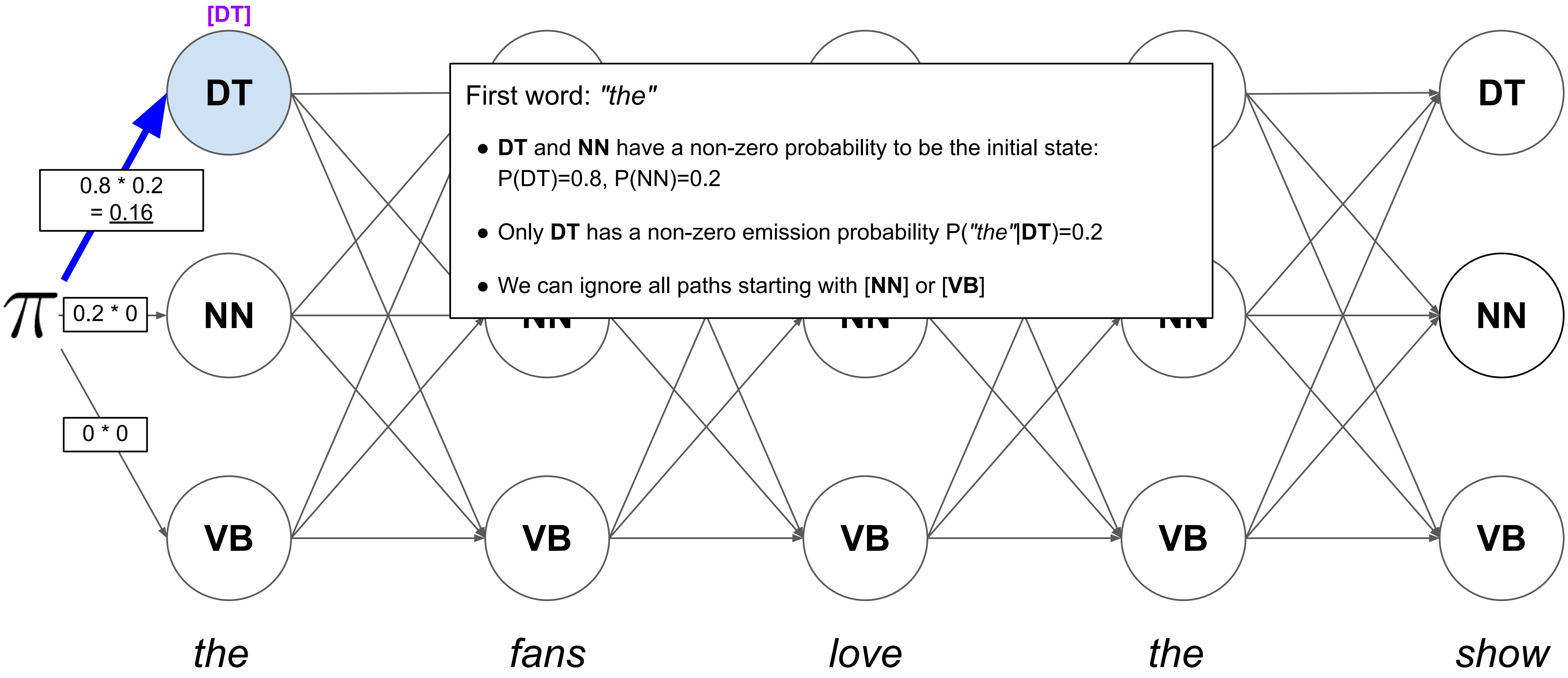
$$B = \begin{array}{cc} \begin{array}{cccc} \textit{the} & \textit{fans} & \textit{love} & \textit{show} \end{array} & \\ \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.0 & 0.05 & 0.3 & 0.1 \\ 0 & 0.25 & 0.15 & 0.3 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$



First word: *"the"*

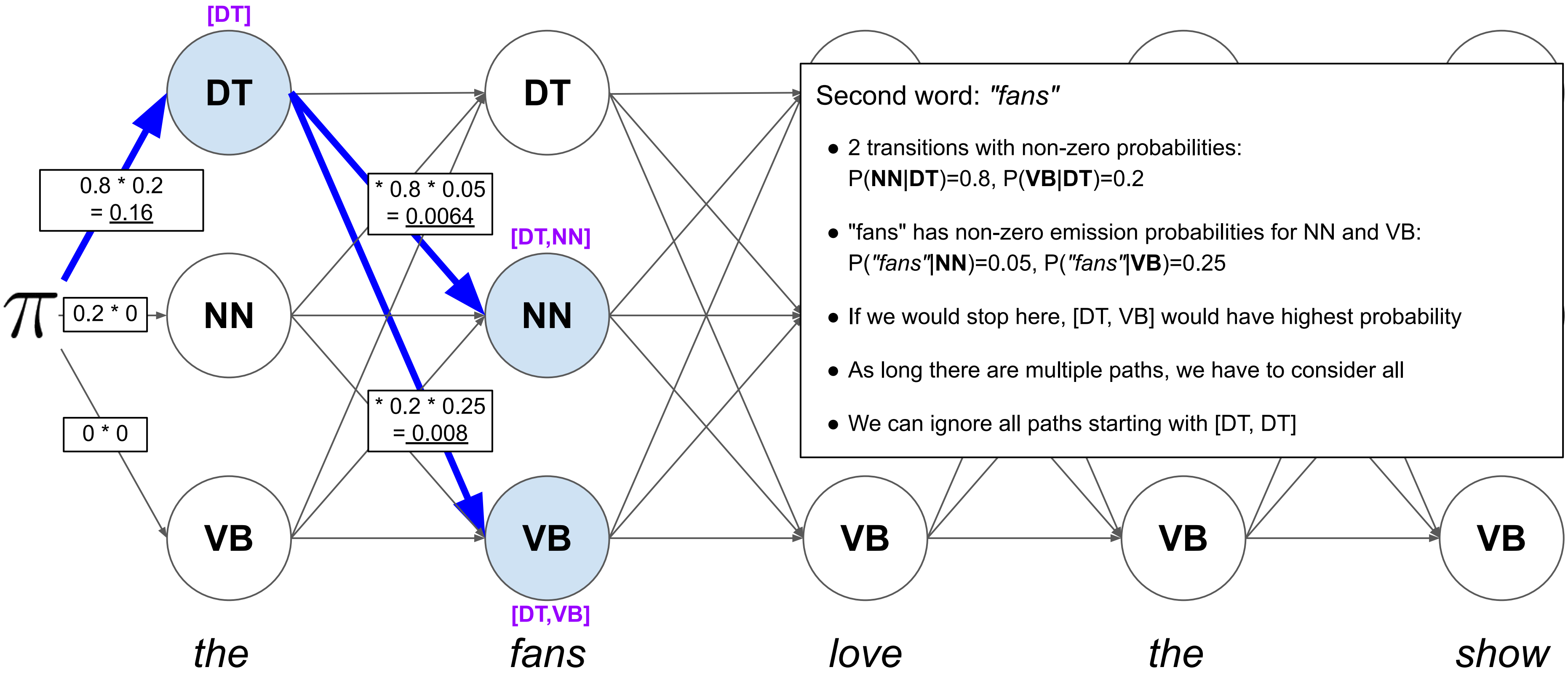
- **DT** and **NN** have a non-zero probability to be the initial state: P(DT)=0.8, P(NN)=0.2
- Only **DT** has a non-zero emission probability P(*"the"*|**DT**)=0.2
- We can ignore all paths starting with [**NN**] or [**VB**]

Example

$$\pi = \begin{array}{c} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

$$A = \begin{array}{cc} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} & \\ \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$

$$B = \begin{array}{cc} \begin{array}{cccc} \textit{the} & \textit{fans} & \textit{love} & \textit{show} \end{array} & \\ \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.0 & 0.05 & 0.3 & 0.1 \\ 0 & 0.25 & 0.15 & 0.3 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$



Second word: *"fans"*

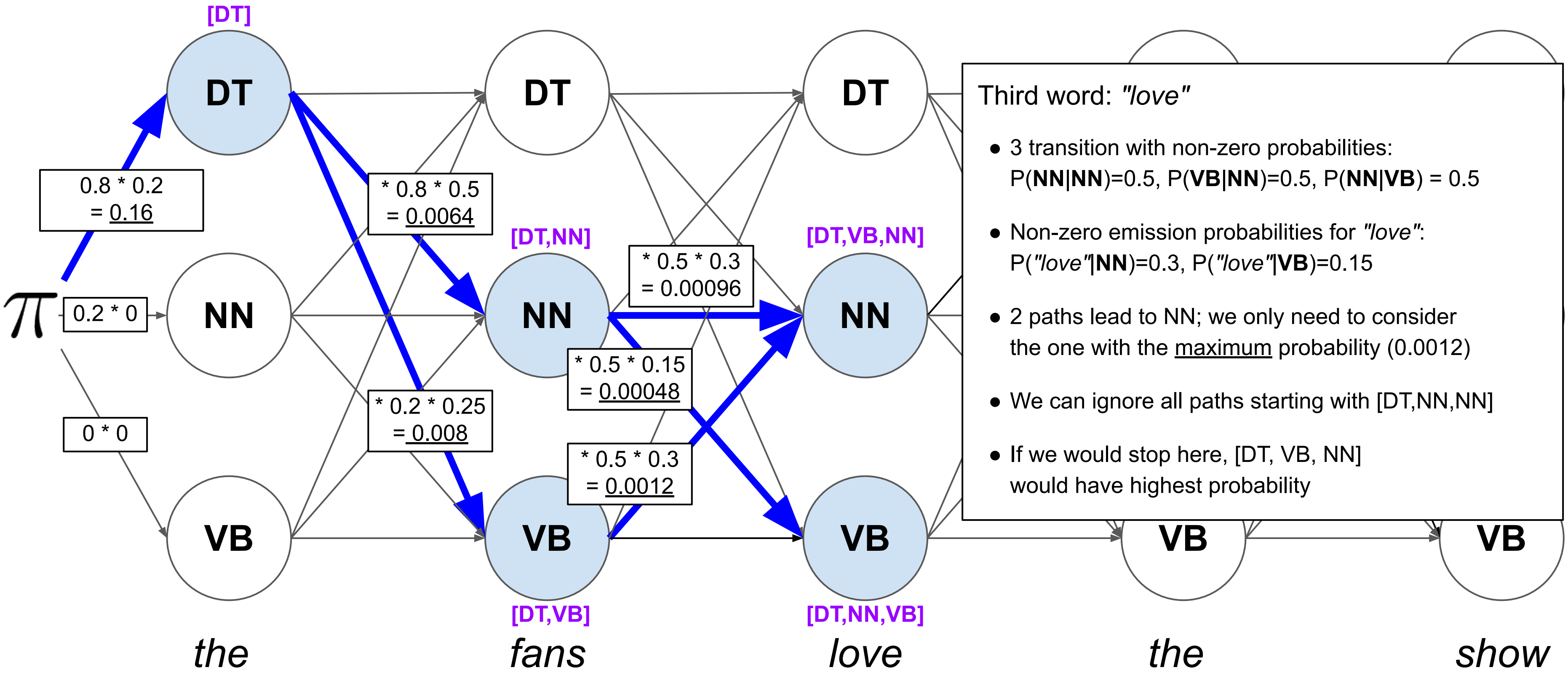
- 2 transitions with non-zero probabilities:
  P(**NN**|**DT**)=0.8, P(**VB**|**DT**)=0.2
- "fans" has non-zero emission probabilities for NN and VB:
  P(*"fans"*|**NN**)=0.05, P(*"fans"*|**VB**)=0.25
- If we would stop here, [DT, VB] would have highest probability
- As long there are multiple paths, we have to consider all
- We can ignore all paths starting with [DT, DT]

Example

$$\pi = \begin{array}{c} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

$$A = \begin{array}{cc} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} & \\ \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$

$$B = \begin{array}{cc} \begin{array}{cccc} \textit{the} & \textit{fans} & \textit{love} & \textit{show} \end{array} & \\ \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.0 & 0.05 & 0.3 & 0.1 \\ 0 & 0.25 & 0.15 & 0.3 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$



Third word: *"love"*

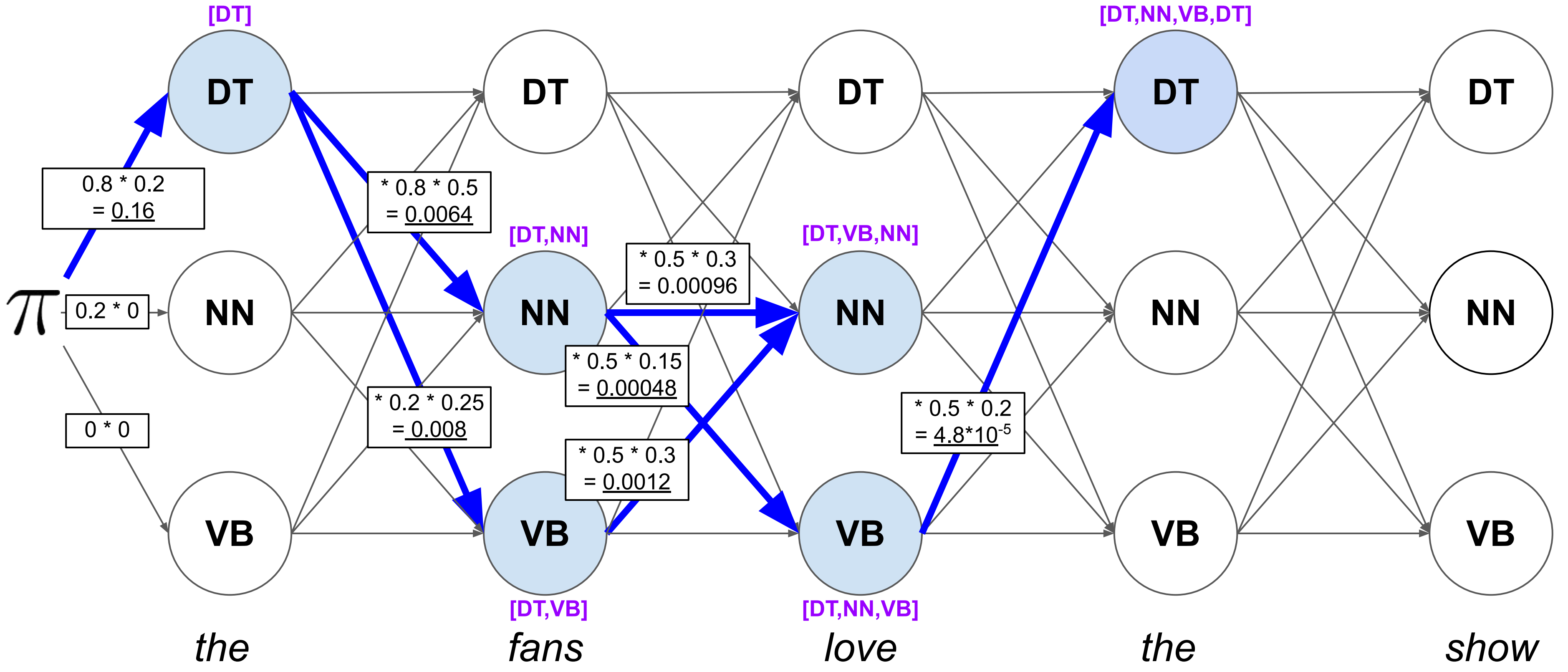
- 3 transition with non-zero probabilities: P(**NN|NN**)=0.5, P(**VB|NN**)=0.5, P(**NN|VB**) = 0.5
- Non-zero emission probabilities for *"love"*: P(*"love"*|**NN**)=0.3, P(*"love"*|**VB**)=0.15
- 2 paths lead to NN; we only need to consider the one with the maximum probability (0.0012)
- We can ignore all paths starting with [DT,NN,NN]
- If we would stop here, [DT, VB, NN] would have highest probability

Example

$$\pi = \begin{array}{c} \begin{matrix} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{matrix} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

$$A = \begin{array}{cc} \begin{matrix} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{matrix} & \\ \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} & \begin{matrix} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{matrix} \end{array}$$

$$B = \begin{array}{cc} \begin{matrix} \textit{the} & \textit{fans} & \textit{love} & \textit{show} \end{matrix} & \\ \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.0 & 0.05 & 0.3 & 0.1 \\ 0 & 0.25 & 0.15 & 0.3 \end{bmatrix} & \begin{matrix} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{matrix} \end{array}$$



Example

$$\pi = \begin{array}{c} \textbf{DT}\ \ \textbf{NN}\ \ \textbf{VB} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

$$A = \begin{array}{cc} \begin{array}{ccc} \textbf{DT} & \textbf{NN} & \textbf{VB} \end{array} & \\ \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} & \begin{array}{c} \textbf{DT} \\ \textbf{NN} \\ \textbf{VB} \end{array} \end{array}$$

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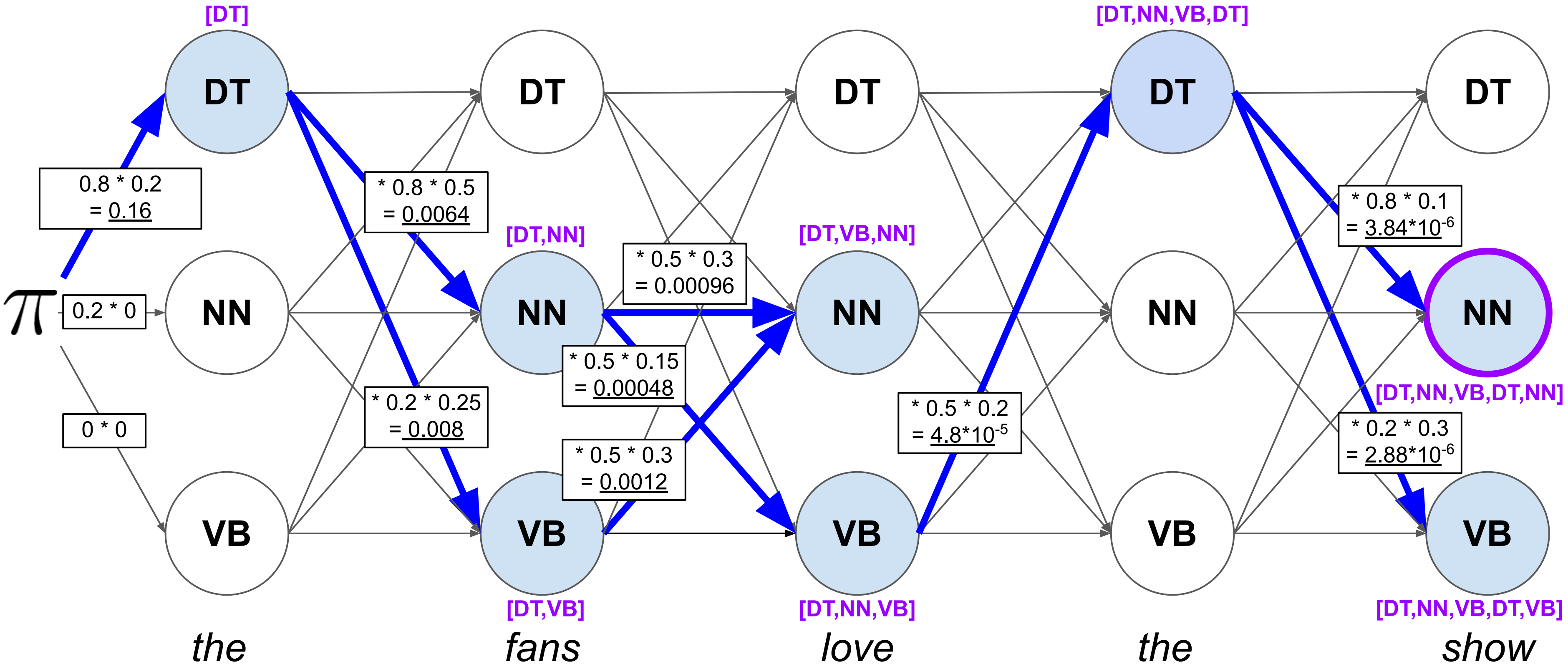
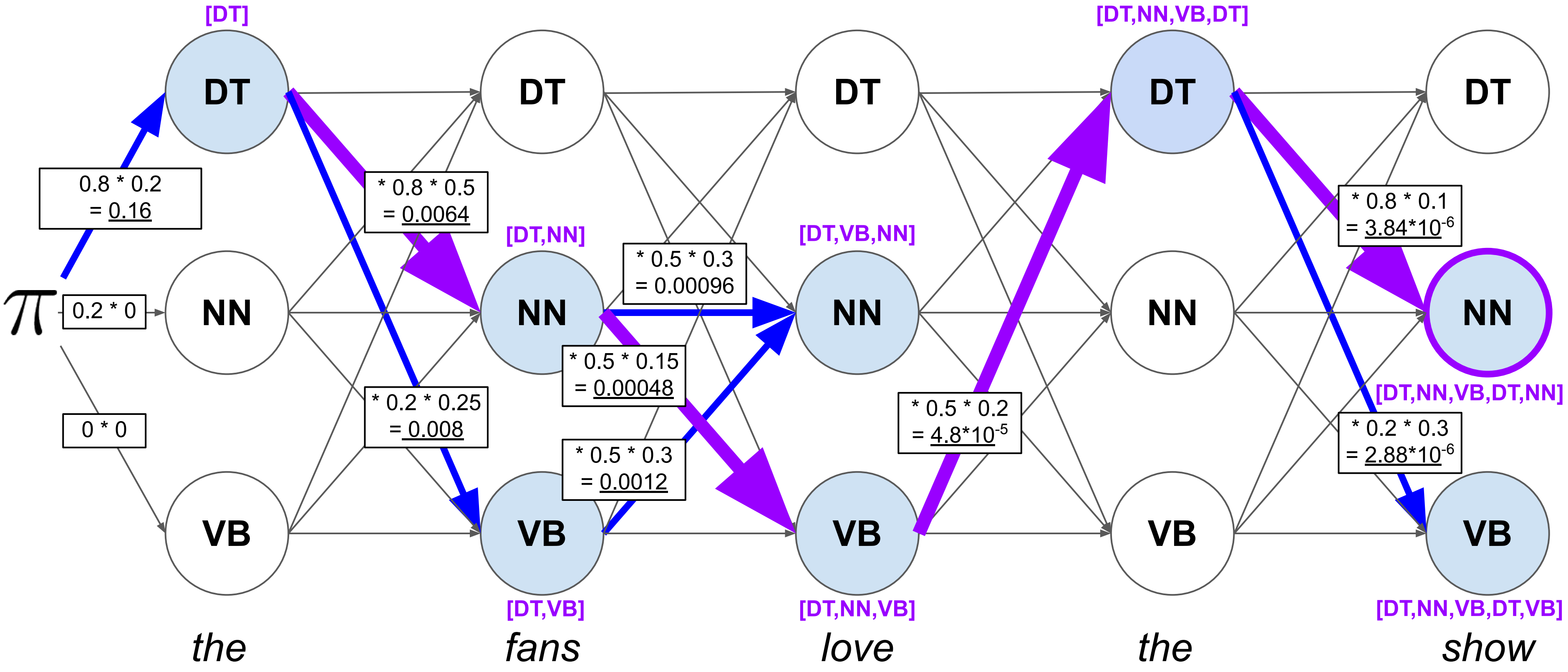


Viterbi Algorithm

- 2 important question
  - How to get the final state sequence with the highest probability?
  - How exactly does the Viterbi algorithm reduces complexity?

Backtracking

- During forward pass: remember input path with max probability
- Backtracking: follow paths with max probabilities back to beginning



Quick Quiz

What is the **runtime complexity** of the **Viterbi** algorithm?

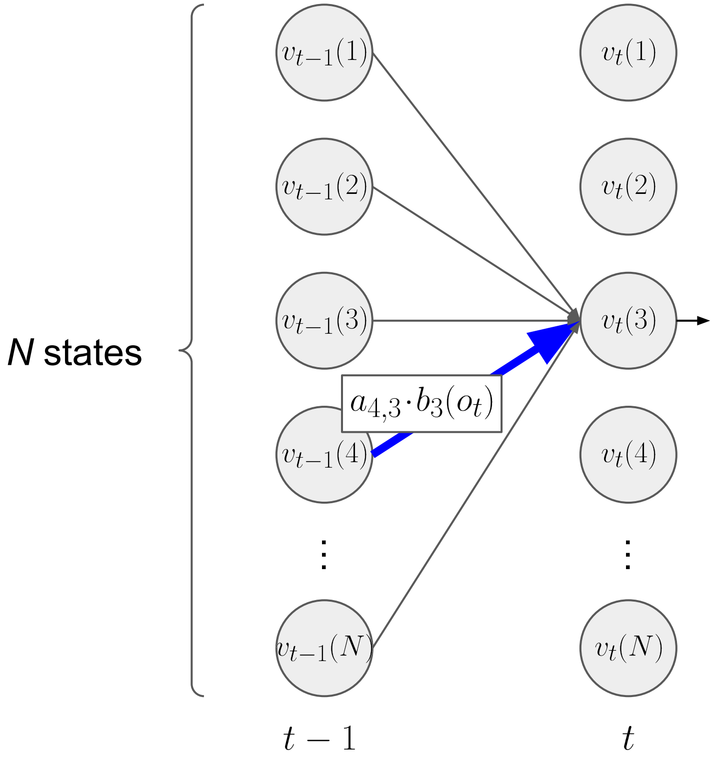
A $O(N + T)$

B $O(N \cdot T)$

C $O(N^2 \cdot T)$

D $O(N^2 \cdot T^2)$

Viterbi Algorithm — Complexity Analysis

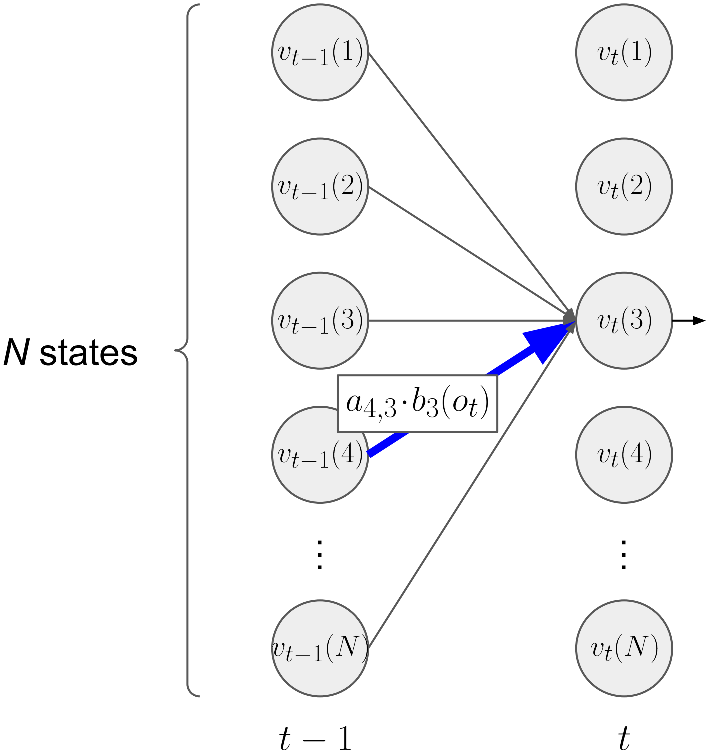


- Let $v_t(3)$ be maximal for the path coming from $v_{t-1}(4)$
- We can ignore all paths coming from $v_{t-1}(j), \quad j \neq 4$
- This holds true for all steps t and states j

➜ Time complexity for Viterbi: $O(\underbrace{T}_{\text{length of sequence}} \cdot \underbrace{N}_{\text{\#states}}{}^2)$

Note: Cases where $a_{ij} \cdot b_j(o_t) = 0$ might be an additional convenience but not the main reason for the polynomial complexity

Viterbi Algorithm — The Basic Algorithm



Initialization

$$v_1(t) = \pi_j \cdot b_j(o_1)$$

$$bt_1(t) = 0$$

$$1 \leq j \leq N$$

Recursion

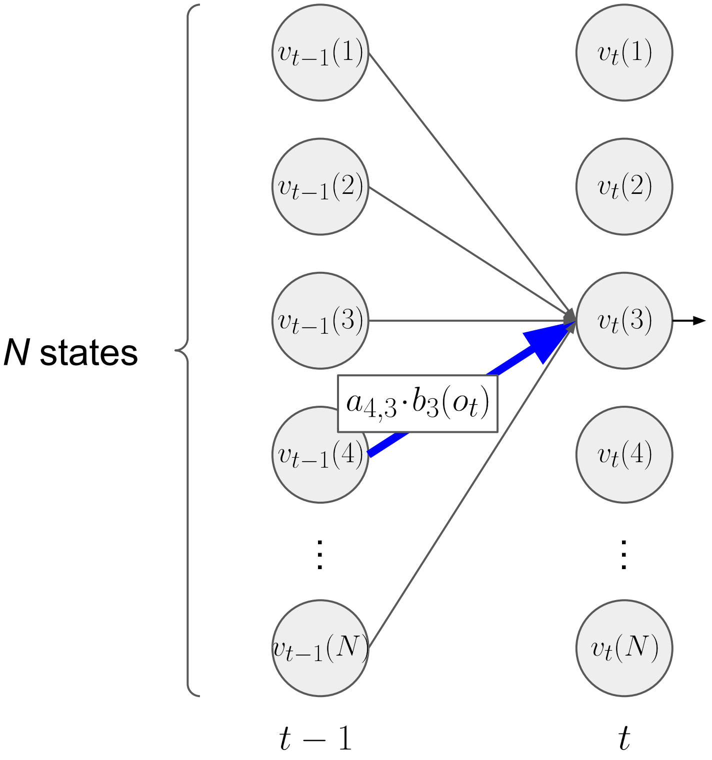
$$v_t(j) = \max_{i=1}^{N} v_{t-1}(i) a_{ij} b_j(o_t)$$

$$bt_t(j) = \operatorname*{argmax}_{i=1}^{N} v_{t-1}(i) a_{ij} b_j(o_t)$$

$$1 \leq j \leq N, \; 1 < t \leq T$$

Example for backtrace: $bt_t(3) = 4$ since we get the highest probability for $v_t(3)$ from the path coming from $v_{t-1}(4)$

Viterbi Algorithm — The Basic Algorithm



Termination (after computing all $v_t(j)$ and $bt_t(j)$)

Probability of most likely path: $P^* = \max_{i=1}^{N} v_T(i)$

Start of backtrace: $q_T^* = \operatorname{argmax}_{i=1}^{N} v_T(i)$

Viterbi Algorithm — Practical Consideration

- The "usual" problem: Risk of arithmetic underflow

$$v_1(t) = \pi_j \cdot b_j(o_1)$$

$$v_t(j) = \max_{i=1}^{N} v_{t-1}(i) a_{ij} b_j(o_t)$$

Values for $v_t(j)$ become very small as we multiple many (potentially very) small probability values

➜ The "usual" solution: Logarithm

$$v_1(t) = \log \pi_j + \log b_j(o_1)$$

$$v_t(j) = \max_{i=1}^{N} v_{t-1}(i) + \log a_{ij} + \log b_j(o_t)$$

Viterbi Algorithm — Python/NumPy Implementation

```
def viterbi(tokens, A, B, PI):
    N, T = A.shape[0], len(tokens)
    M = np.zeros((N, T))                         # Reflecting probabilties of trellis
    BT = np.zeros((N, T), dtype=np.int16)  # For the Backtracking pointers

    # Initialization
    for s in range(N):
        M[s,0] = PI[s] * B[s, word2index[tokens[0]]]

    # Recursion (with dynamic programming)
    for t in range(1, T):
        for s in range(N):
            new_probs = M[:,t-1] * A[:,s] * B[s, word2index[tokens[t]]]
            max_idx = np.argmax(new_probs)
            M[s,t]  = new_probs[max_idx]
            BT[s,t] = max_idx

    # Termination (start backtracking)
    state = np.argmax(M[:,-1])
    state_sequence = []
    for i in reversed(range(T)):
        state_sequence.append(state)
        state = BT[:,i][state]

    return [ index2tag[idx] for idx in reversed(state_sequence) ]
```

Note: This slide is only to show that it does not take much code to implement the Viterbi algorithm.

$$v_1(t) = \pi_j \cdot b_j(o_1) \qquad bt_1(t) = 0$$

$$v_t(j) = \max_{i=1}^{N} v_{t-1}(i) a_{ij} b_j(o_t)$$

$$bt_t(j) = \operatorname*{argmax}_{i=1}^{N} v_{t-1}(i) a_{ij} b_j(o_t)$$

$$q_T^* = \operatorname*{argmax}_{i=1}^{N} v_T(i)$$

Viterbi Algorithm — Python/NumPy Implementation

- Using the HMM trained over 25k movie reviews
  - 50 states (POS tags)
  - 83k+ tokens (words, punctuation marks, etc.)

Important: I've cheated here by annotating the reviews using spaCy, not humans!

```
viterbi(['the', 'fans', 'love', 'the', 'show'], A, B, PI)
```

```
['DT', 'NNS', 'VBP', 'DT', 'NN']
```

```
viterbi(['the', 'fans', 'like', 'the', 'show'], A, B, PI)
```

```
['DT', 'NNS', 'IN', 'DT', 'NN']
```

```
viterbi(['funny', 'movies', 'are', 'the', 'best'], A, B, PI)
```

```
['JJ', 'NNS', 'VBP', 'DT', 'JJS']
```

```
viterbi(['i', 'like', 'watching', 'comedies'], A, B, PI)
```

```
['PRP', 'VBP', 'VBG', 'NNS']
```

Outline

- Overview: Sequence Tasks
- POS Tagging
  - What are Parts of Speech?
  - Why is this task important and challenging?
- Hidden Markov Models (HMM)
  - Basic setup and components
  - Core HMM tasks
    - Model Learning
    - Likelihood computation
    - Viterbi decoding

Summary

- Sequences
  - A primary form of natural language data with many applications
    (a sentence is sequence of words; sequence captures meaning ➜ BoW model intrinsically limited)
  - Many sequence tasks in NLP
- Focus of this lecture: sequence labeling
  - POS tagging as very fundamental sequence labeling task
  - Different approaches, incl. Hidden Markov Models (HMM)
- Next lecture: encoder-decoder architecture
  - Neural network-based architecture
  - Applicable to all sequence tasks

Pre-Lecture Activity for Next Week

- Assigned Task
  - Post a 1-2 sentence answer to the following questions into the Discussion forum

"What is an encoder? What is a decoder?
What are we trying to encode/decode anyways?"

Solutions to Quick Quizzes

- Slide 15: D hopefully :)
- Slide 18: B
  - "up" can be: noun, verb, adverb, adjective, preposition, adjective
- Slide 22: B
  - All other words can be a noun or a verb
- Slide 30: A
  - Generate random state sequences based on transition probabilities
  - Generate random observations for each state based on emission probabilities
- Slide 35
  - Particularly for the emission probabilities we might see a word for the first time
  - Zero-counts lead to zero-probabilities which generally skew the results
  - Basic counter-measure: smoothing

Solutions to Quick Quizzes

- Slide 43: A
  - N^T is the number of possible paths / number of possible combinations of states
  - For each path / state combination we have to compute the likelihood, which is in $O(T)$
- Slide 55: C
  - See Slide 56 for explanation