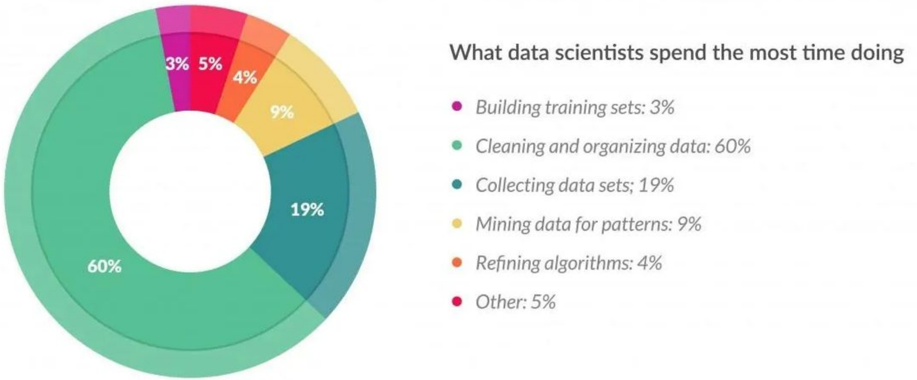
NUS | Computing
National University of Singapore

CS4248: Natural Language Processing

Lecture 3 — n-Gram Language Models

What do you want to learn?

- *"Understanding LLMs such ~~ChatGPT"~~*
  - Provocative statement: Nobody really understands LLMs, i.e., why/how they work!
  - The way to understand LLMs requires a lot of background which we will cover
  - We end with an introduction into LLMs, but they are not and can not be the focus of CS4248
    (a dedicated graduate course covering LLMs is currently in the planning/preparation stage – stay tuned!)
  - In practice, fine-tuning LLMs is much more about proper data preparation than the actual training



Source: https://www.forbes.com/sites/gilpress/2016/03/23/data-preparation-most-time-consuming-least-enjoyable-data-science-task-survey-says/

What do you want to learn?

- *HuggingFace, Langchain, Tensorflow, PyTorch, scikit-learn, numpy, etc.*
  - The lecture content focusing on the fundamental concept, not specific tools and libraries
  - We provide many practical examples in our series supplementary notebooks
  - You are free and encouraged to explore any available tools/frameworks/libraries for your project

Your Concerns — Our Comments

- *"I'm a total NLP/ML noob"*
  - CS4248 is a introduction / foundation course – we basically start from scratch
  - While some background knowledge is certainly useful, it's not a requirement
  - We only focus on nitty-gritty details required for this course (e.g., we do not cover backpropagation)

- *"I'm worried that there will be lots of math."*
  - Yes, there will be math, but nothing beyond <u>fundamental</u> concepts of algebra, probability, calculus
  - What need we need in this course, you will need in <u>any</u> computer/data science field!
  - We hope we cover the math bits in sufficient detail and clarity (if not, you can always ask!)

Your Concerns — Our Comments

- *"I've heard this course is hard!", "I'm afraid of the workload."*
  - Bias alert: We don't think that CS4248 is harder (or easier) than other courses
  - The assessment components are very similar to other course – participation marks are basically free marks :)
  - Consider assignments not just as an assessment component but as a learning experience

- *"I'm worried about the project."*
  - With reasonable effort, it is almost impossible to "fail" the project – we don't expect SOTA results :)
  - Basic suggestions: start early, continuous progress, regular team meetings (and/or with TA)
  - The project provides some flexibility to cater your background and interests
  - You can and should raise any inter-group conflicts incl. non-contributing members
    (there will be 2 rounds of peer review sessions using TEAMMATES!)

Recap of Week 02

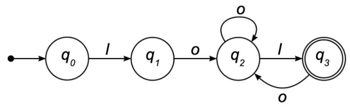
Relationship to Finite State Automata

- Equivalence
  - Regular Expressions describe **Regular Languages** (most restricted types of languages with respect to the Chomsky Hierarchy)
  - Regular Language = language accepted by a FSA

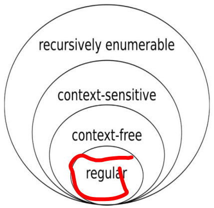
Regular Expression
l(o+l)+

Regular Language
{lol, loool, lolol, looolol, …}

Example: FSA that accepts the Regular Language described by the Regular Expression **l(o+l)+**



Chomsky Hierarchy
(Source: Wikipedia)



Minimum Edit Distance — Backtrace & Alignments

E	8	↙←↓ 9	↓ 8	↓ 7	↙←↓ 8	↓ 7	↓ 6	↙ 5
G	7	↙←↓ 8	↓ 7	↓ 6	↙←↓ 7	↓ 6	↙ 5	← 6
A	6	↙←↓ 7	↙↓ 6	↓ 5	↙←↓ 6	↙ 5	← 6	← 7
U	5	↙←↓ 6	↓ 5	↙ 4	← 5	← 6	←↓ 7	↙←↓ 8
G	4	↙←↓ 5	↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙ 6	← 7
N	3	↙←↓ 4	↓ 3	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8
A	2	↙←↓ 3	↙ 2	← 3	← 4	↙← 5	← 6	← 7
L	1	↙←↓ 2	↙←↓ 3	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8
#	0	1	2	3	4	5	6	7
	#	S	A	U	S	A	G	E

Quick quiz: Why do we choose the diagonal path here?

L A N G U * A G E
| | | | | | | | |
S A * * U S A G E

Complexity analysis
Time: O(n+m)

Tokenization — BPE Token Learner

corpus representation

6	n e w e s t _
5	l o w _
3	w i d e s t _
2	l o w e r _
1	l o n g e r _

vocabulary
d, e, g, i, l, n, o, r, s, t, w, _

merges

most frequent pair: **e** & **s** (9 occurrences)

corpus representation

6	n e w **es** t _
5	l o w _
3	w i d **es** t _
2	l o w e r _
1	l o n g e r _

vocabulary
d, e, g, i, l, n, o, r, s, t, w, _, **es**

merges
(e, s)

most frequent pair: **es** & **t** (9 occurrences)

Noisy Channel Model — Calculating/Estimating $P(x|w)$

$$P(x|w) = \begin{cases} \frac{ins[w_{i-1}, x_i]}{count[w_i]}, & \text{if insertion} \\ \frac{del[w_{i-1}, w_i]}{count[w_{i-1}, w_i]}, & \text{if deletion} \\ \frac{sub[x_i, w_i]}{count[w_i]}, & \text{if substitution} \\ \frac{trans[w_i, w_{i+1}]}{count[w_i, w_{i+1}]}, & \text{if transposition} \end{cases}$$

w_i = i-th character in the correct word w

x_i = i-th character in the misspelled word x

Outline

- **Language Models**
  - **Motivation**
  - Sentence Probabilities
  - Markov Assumption
  - Challenges
- Smoothing
  - Laplace Smoothing
  - Backoff & Interpolation
  - Kneser-Ney Smoothing
- Evaluating Language Models

Pre-Lecture Activity from Last Week

- Assigned Task
  - Post a 1–2 sentence answer to the following question into the L1 Discussion
    (you will find the thread on Canvas > Discussions)

“What do we mean when we talk about the probability of a sentence?”

Side notes:
- This task is meant as a warm-up to provide some context for the next lecture
- No worries if you get lost; we will talk about this in the next lecture
- You can just copy-&-paste others' answers, but his won't help you learn better

The probability that a sequence of words will form a coherent sentence with the correct context given prior knowledge.

How likely it is a sentence is grammatically correct.

Perhaps the probability that the sentence has a certain meaning?

LH

It means that the probability that the sentence is a valid or natural expression in a given language.

P(sentence) = 1 / # all possible sentences

It is the product of the probability of its words.

SH

The likelihood that a particular sequence of words forms a grammatically correct and meaningful statement within a given language.

It means how likely a sentence/phrase is expected to appear.

For example, if one of the training examples is 'I like cats', then 'cats' has a possibility to appear after 'I like' appears.

It refers to a sequence of words, and the probability that a word appears given the previous word in a sentence. Subsequently, all these successive probabilities can be calculated using chain rule to calculate the join probability of all word sequences to find the final sentence

The probability of a sentence is the probability that this sequence of words will appear given a random collection of words. For instance, the probability of the sentence "I am hungry" is P('I') * P('am' | 'I') * P('hungry' | 'I am').

Language Models — Motivation

- Which sentence makes more sense? S_1 or S_2?

Example 1:

S_1: *"on guys all I of noticed sidewalk three a sudden standing the"*

S_2: *"all of a sudden I noticed three guys standing on the sidewalk"*

Example 2:

S_1: *"the role was played by an ~~**acress**~~**across** famous for her comedic timing"*

S_2: *"the role was played by an ~~**acress**~~**actress** famous for her comedic timing"*

- But why?
  - Probability of S_2 higher than of S_1: $P(S_2) > P(S_1)$

➜ **Language Models** — Assigning probabilities to a sentence, phrase (or word)

Language Models — Basic Idea

- 2 basic notions of probabilities

(1) Probability of a sequence of words W

$$P(W) = \boxed{P(w_1, w_2, w_3, \ldots, w_n)}$$

Joint

Example: $P(\text{“}remember\ to\ submit\ your\ assignment\text{”})$

(2) Probability of an upcoming word w_n

Conditional

$$P(w_n \mid w_1, w_2, w_3, \ldots, w_{n-1})$$

Example: $P(\text{“}assignment\text{”} \mid \text{“}remember\ to\ submit\ your\text{”})$

In this lecture: How to calculate these probabilities?

Language Models — Applications

- Language Models are fundamental for many NLP tasks
  - **Speech Recognition** $P(\text{“}we\ built\ this\ city\ on\ rock\ and\ roll\text{”}) > P(\text{“}we\ built\ this\ city\ on\ sausage\ rolls\text{”})$
  - **Spelling correction** $P(\text{“}\ldots\ has\ no\ mistakes\text{”}) > P(\text{“}\ldots\ has\ no\ \underline{mistaek}\text{”})$
  - **Grammar correction** $P(\text{“}\ldots\ has\ improved\text{”}) > P(\text{“}\ldots\ has\ improve\text{”})$
  - **Machine Translation** $P(\text{“}I\ went\ home\text{”}) > P(\text{“}I\ went\ to\ home\text{”})$

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Probabilities of Sentences (more generally: sequence of words)

$$P(\text{“}remember\ to\ submit\ your\ assignment\text{”})$$
$$P(\text{“}assignment\text{”}\,|\,\text{“}remember\ to\ submit\ your\text{”})$$

➜ How to calculate those probabilities?

$P(A_1, A_2) \cdot P(A_2)$

- Quick review: Chain Rule (allows the iterative calculation of joint probabilities)
  - Chain rule for 2 random events: $\boxed{P(A_1, A_2)} = P(A_2|A_1) \cdot P(A_1)$
  - Chain rule for 3 random events: $$\begin{aligned} P(A_1, A_2, A_3) &= P(A_3|A_1, A_2) \cdot P(A_1, A_2) \\ &= P(A_3|A_1, A_2) \cdot P(A_2|A_1) \cdot P(A_1) \end{aligned}$$
  - …

Probabilities of Sentences

- Chain rule — generalization to *N* random events

$$P(A_1, \ldots, A_N) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_{1:\,2}) \cdot \cdots \cdot P(A_N|A_{1:\,N-1})$$
$$= \prod_{i=1}^{N} P(A_i|A_{1:\,i-1})$$

$i:j$ — sequence notations

➜ Chain rule applied to sequences of words

$$P(w_1, \ldots, w_N) = P(w_1) \cdot P(w_2|w_1) \cdot P(w_3|w_{1:\,2}) \cdot \ldots \cdot P(w_N|w_{1:\,N-1})$$
$$= \prod_{i=1}^{N} P(w_i|w_{1:\,i-1})$$

🏃‍♀️🏃🏃‍♂️ Probably Correct? (5 mins)

Given two random variables X and Y with known probabilities $P(X)$ and $P(Y)$, compose as many statements with the tokens:

$$P(X) \quad P(Y) \quad P(Y|X) \quad P(X|Y) \quad > \quad < \quad =$$

And classify them as always correct, sometimes correct or never correct.

Post your answer to Canvas > Discussions > [In-Lecture Interaction] L1

(One student of your group can post the reply, and make sure to include your group members' names)

Probabilities of Sentences

- Calculating the probabilities using Maximum Likelihood Estimations

$$P(w_n|w_{1:n-1}) = \frac{Count(w_{1:n-1}w_n)}{\sum_w Count(w_{1:n-1}w)} = \boxed{\frac{Count(w_{1:n})}{Count(w_{1:n-1})}}$$

Quick quiz: Why does the denominator simplify like this?

Probabilities of Sentences — Example

(1) Application of Chain Rule

$$P(\text{"remember to submit your assignment"}) = \begin{aligned} &P(\text{"remember"}) \cdot \\ &P(\text{"to"} \mid \text{"remember"}) \cdot \\ &P(\text{"submit"} \mid \text{"remember to"}) \cdot \\ &P(\text{"your"} \mid \text{"remember to submit"}) \cdot \\ &P(\text{"assignment"} \mid \text{"remember to submit your"}) \end{aligned}$$

(2) Maximum Likelihood Estimation

$$P(\text{"remember"}) = \frac{Count(\text{"remember"})}{N}$$

$$P(\text{"to"} \mid \text{"remember"}) = \frac{Count(\text{"remember to"})}{Count(\text{"remember"})}$$

$$\cdots$$

$$P(\text{"assignment"} \mid \text{"remember to submit your"}) = \frac{Count(\text{"remember to submit your assignment"})}{Count(\text{"remember to submit your"})}$$

🤔 **Foreshadowing**:
Do you see any problems?

Probabilities of Sentences — Problems

$V = 30K$

$$P(\text{“}assignment\text{”} \mid \text{“}remember\ to\ submit\ your\text{”}) = \frac{Count(\text{“}remember\ to\ submit\ your\ assignment\text{”})}{Count(\text{“}remember\ to\ submit\ your\text{”})}$$

- Problem: (very) long sequences
  - Large number of entries in table with joint probabilities
  - A sequence (or subsequence) $w_{i:j}$ may not be present in corpus $\Bigg\}$ ➜ $Count(w_{i:j}) = 0$ ➜ $\prod_{n=1}^{N} P(w_n | w_{1:\,n-1}) = 0$

(we can ignore $\frac{0}{0}$ here; this can be handled in the implementation)

➜ Can we keep the sequences short?

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  - **Markov Assumption**
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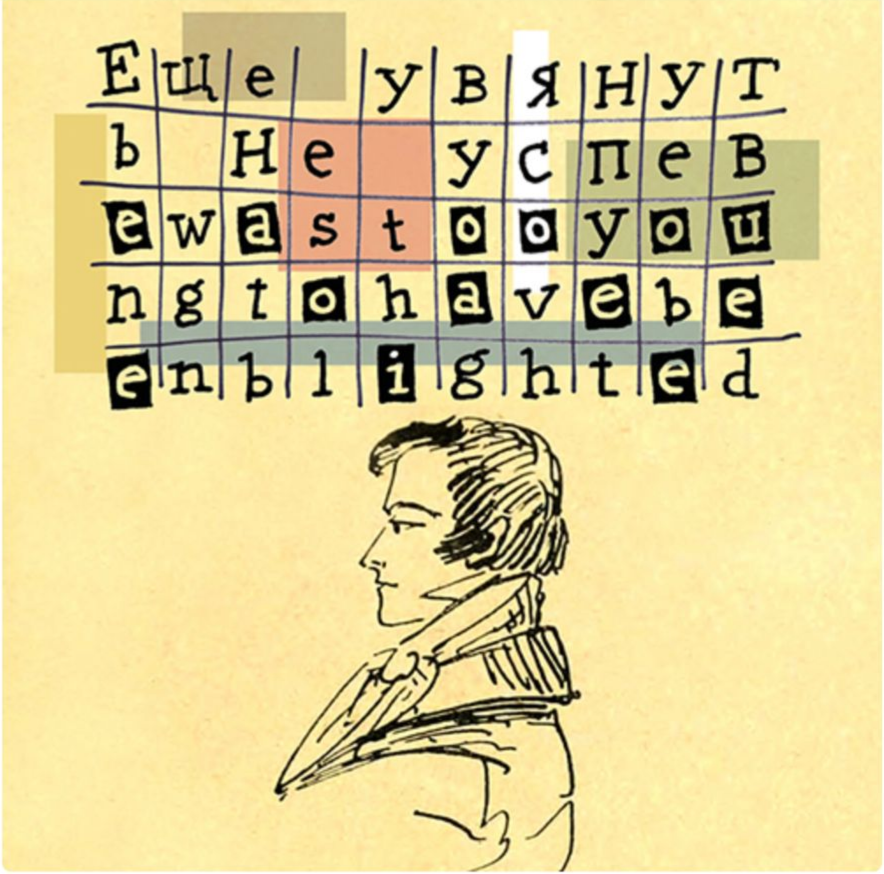
AMERICAN Scientist

“The first application of [A. A. Markov’s chains] was to a textual analysis of Alexander Pushkin's poem Eugene Onegin. Here a snippet of one verse appears (in Russian and English) along with Pushkin's own sketch of his protagonist Onegin.”



Markov Assumption

- Probabilities depend on only on the last k words

$$P(w_1, \ldots, w_N) = \prod_{n=1}^{N} P(w_n | w_{1:n-1}) = \prod_{n=1}^{N} P(w_n | w_{n-k:n-1})$$

- For our example:

$$P(\text{“assignment”} \mid \text{“remember to submit your”}) \approx P(\text{“assignment”} \mid \text{“your”})$$

$$P(\text{“assignment”} \mid \text{“submit your”})$$

$$P(\text{“assignment”} \mid \text{“to submit your”})$$

...

n-Gram Models (consider the only *n-1* last words)

exact ≮

Unigram (1-gram): $P(w_n|w_{1:\,n-1}) \approx ???$ $P(w_n)$

Bigram (2-gram): $P(w_n|w_{1:\,n-1}) \approx ???$ $P(w_n | w_{n-1})$

Trigram (3-gram): $P(w_n|w_{1:\,n-1}) \approx ???$ $P(w_n | w_{n-1}, w_{n-2})$

n-Gram Models (consider the only *n-1* last words)

Unigram (1-gram): $P(w_n|w_{1:\,n-1}) \approx P(w_n)$

Bigram (2-gram): $P(w_n|w_{1:\,n-1}) \approx P(w_n|w_{n-1})$

Trigram (3-gram): $P(w_n|w_{1:\,n-1}) \approx P(w_n|w_{n-2}, w_{n-1})$

Quick quiz: How does this relate to context-sensitive or context-free?

n-Gram Models

V = 30K

Maximum Likelihood Estimation

Unigram (1-gram): $P(w_n|w_{1:\,n-1}) \approx P(w_n)$ $\qquad P(w_n) = \dfrac{Count(w_n)}{\#\ words}$

Bigram (2-gram): $P(w_n|w_{1:\,n-1}) \approx P(w_n|w_{n-1})$ $\qquad P(w_n|w_{n-1}) = \dfrac{Count(w_{n-1}w_n)}{Count(w_{n-1})}$

Trigram (3-gram): $P(w_n|w_{1:\,n-1}) \approx P(w_n|w_{n-2}, w_{n-1})$ $\qquad P(w_n|w_{n-1}, w_{n-2}) = \dfrac{Count(w_{n-2}w_{n-1}w_n)}{Count(w_{n-2}w_{n-1})}$

Accuracy ↑

General MLE for *n*-grams: $P(w_i|w_{n-N+1:n-1}) = \dfrac{Count(w_{n-N+1:i})}{Count(w_{n-N+1:n-1})}$

- n-Gram models in practice
  - 3-gram, 4-gram, 5-gram models very common
  - The larger the n-grams, the more data required

🤔 **To Think About**:
How much more data?

n-Gram Models — Bigram Example

Example corpus with 3 sentences

<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>

$$P(\text{“}I\text{”} \mid \text{“}<s>\text{”}) = \frac{Count(\text{“}<s>\ I\text{”})}{Count(\text{“}<s>\text{”})} =$$

$$P(\text{“}am\text{”} \mid \text{“}I\text{”}) = \frac{Count(\text{“}I\ am\text{”})}{Count(\text{“}I\text{”})} =$$

$$P(\text{“}Sam\text{”} \mid \text{“}am\text{”}) = \frac{\boxed{Count(\text{“}am\ Sam\text{”})}}{\boxed{Count(\text{“}am\text{”})}} =$$

$$P(\text{“}</s>\text{”} \mid \text{“}Sam\text{”}) = \frac{Count(\text{“}Sam\ </s>\text{”})}{Count(\text{“}Sam\text{”})} =$$

n-Gram Models — Bigram Example

Example corpus with 3 sentences

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$$P(\text{“}I\text{”}|\text{“}<s>\text{”}) = \frac{Count(\text{“}<s>\ I\text{”})}{Count(\text{“}<s>\text{”})} = \frac{2}{3}$$

$$P(\text{“}am\text{”}|\text{”}I\text{”}) = \frac{\boxed{Count(\text{“}I\ am\text{”})}}{\boxed{Count(\text{“}I\text{”})}} = \frac{2}{3}$$

$$P(\text{“}Sam\text{”}|\text{”}am\text{”}) = \frac{Count(\text{“}am\ Sam\text{”})}{Count(\text{“}am\text{”})} = \frac{1}{2}$$

$$P(\text{“}</s>\text{”}|\text{“}Sam\text{”}) = \frac{Count(\text{“}Sam\ </s>\text{”})}{Count(\text{“}Sam\text{”})} = \frac{1}{2}$$

n-Gram Models — Bigram Example (25,000 Movie Reviews)

$$P(\text{“}<s>\ \ i\ like\ the\ story\ \ </s>\text{”}) = ???$$

Unigram counts:

i	like	the	story
87,185	19,862	33,0867	11,094

Bigram counts:

	i	like	the	story
i	1	693	20	0
like	326	3	1,997	8
the	15	42	148	5171
story	23	16	16	0

n-Gram Models — Bigram Example (25,000 Movie Reviews)

$$P(\text{“}<s>\ \boxed{i}\ like\ the\ story\ \ </s>\text{”}) = ???$$

Unigram counts:

i	like	the	story
87,185	19,862	33,0867	11,094

Bigram counts: w_i

w_{i-1}

	i	like	the	story
i	0	693	20	0
like	326	0	1,997	8
the	15	42	0	5,171
story	23	16	16	0

Bigram probabilities:

	i	like	the	story
i	0.0	0.007949	0.000229	0.0
like	0.016413	0	0.100544	0.000403
the	0.000045	0.000127	0.0	0.015629
story	0.002073	0.001442	0.001442	0.0

Example calculation:

$$P(\text{“}like\text{”}\,|\,\text{“}i\text{”}) = \frac{Count(\text{“}i\ like\text{”})}{Count(\text{“}i\text{”})} = \boxed{\frac{693}{87185}} = 0.007949$$

n-Gram Models — Bigram Example (25,000 Movie Reviews)

Bigram probabilities:

	i	like	the	story
i	0.0	**0.007949**	0.000229	0.0
like	0.016413	0.0	**0.100544**	0.000403
the	0.000045	0.000127	0.0	**0.015629**
story	0.002073	0.001442	0.001442	0.0

Not in the table:

$$P(\text{“}i\text{”}|\text{“}{<}s{>}\text{”}) = 0.088198$$

$$P(\text{“}{<}/s{>}\text{”}|\text{“}story\text{”}) = 0.001262$$

$$P(\text{“}{<}s{>}\ \ i\ like\ the\ story\ \ {<}/s{>}\text{”}) = P(\text{“}i\text{”}|\text{“}{<}s{>}\text{”}) \cdot P(\text{“}like\text{”}|\text{“}i\text{”}) \cdot P(\text{“}the\text{”}|\text{“}like\text{”}) \cdot P(\text{“}story\text{”}|\text{“}the\text{”}) \cdot P(\text{“}{<}/s{>}\text{”}|\text{“}story\text{”})$$

$$P(\text{“}{<}s{>}\ \ i\ like\ the\ story\ \ {<}/s{>}\text{”}) = 0.088198 \cdot 0.007949 \cdot 0.100544 \cdot 0.015629 \cdot 0.001262$$

$$P(\text{“}{<}s{>}\ \ i\ like\ the\ story\ \ {<}/s{>}\text{”}) \simeq 0.0000000139$$

Quick quiz: Why don't we need $P(\text{“}{<}s{>}\text{”})$?

n-Gram Models — Practical Consideration

- In general
  - Each $P(w_n|w_{1:\,n-1})$ rather small ➜ $\prod_{n=1}^{N} P(w_n|w_{1:\,n-1})$ very small
  - Risk of arithmetic underflow

➜ Always use an equivalent logarithmic format
  - Logarithm is a strictly monotonic function

$$P_1 \cdot P_2 \cdot P_3 \cdot \ldots P_N \propto \log\left(P_1 \cdot P_2 \cdot P_3 \cdot \ldots P_N\right)$$

$$= \log P_1 + \log P_2 + \log P_3 \cdot \ldots \log P_N$$

🏃‍♀️🏃🏃‍♂️ Quick Quiz (2 mins)

$P(w_i)$

Given a **unigram** language model and the following two sentences S_1 and S_2

S_1: *"alice saw the accident"*

S_2: *"the accident alice saw"*

which sentence has the **higher probability**?

A $P(S_1) > P(S_2)$

B $P(S_1) < P(S_2)$

C $P(S_1) = P(S_2)$

D insufficient data

🏃‍♀️🏃🏃‍♂️ In-Lecture Activity (5 mins)

- Task: Calculate the Probability **P("saw"|"alice")** given the table of bigram counts below

- Post your answer to Canvas > Discussions > [In-Lecture] L1 … (Feb 2)
(One student of your group can post the reply. Make sure to include your group members' names)

alice accident	5
saw alice	5
alice the	15
alice saw	20
saw the	25
accident saw	1
accident alice	2

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Handling OOV Words — Closed vs. Open Vocabulary

- Closed vocabulary
  - All strings contain words from a fixed vocabulary

➜ **No unknown words**

- Open Vocabulary
  - Strings may contain words that are not in the vocabulary (**OOV words**)
  - Examples: proper nouns, mismatching context

➜ **Counts might be 0** (even for individual words and not just for long(er) sequences of words)

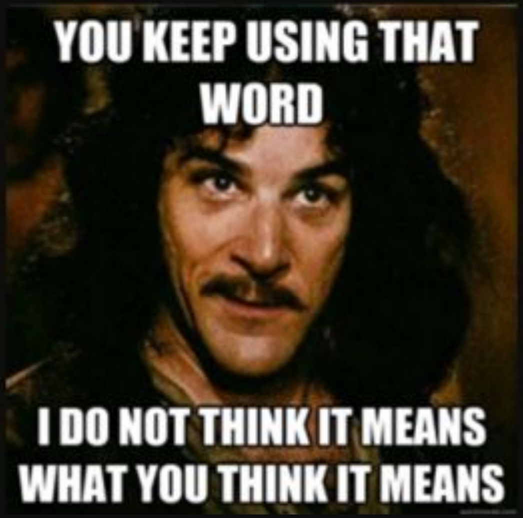
Movie review dataset — Unigram counts:

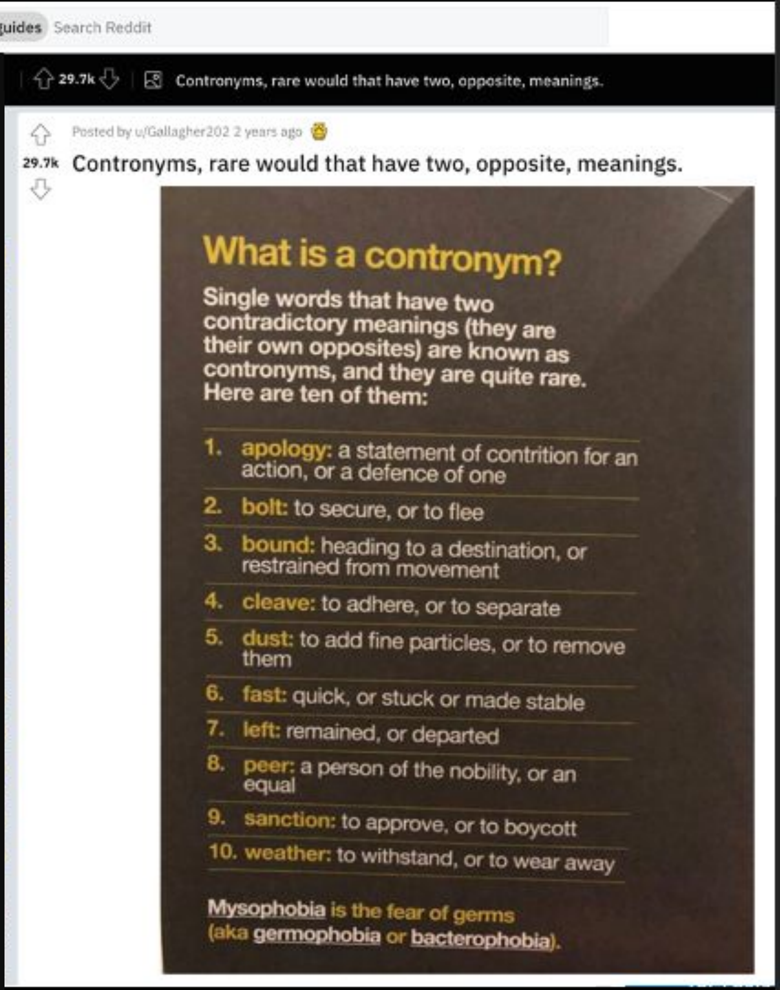
i	like	the	story	costner	einstein	planck	biden	integral	adverb	tensor	nlp
87,185	19,862	33,0867	11,094	67	20	**0**	**0**	27	**0**	**0**	**0**

Handling OOV Words — Alternatives

- Special token for OOV words
  - During normalization, replace all OOV words with a special token (e.g., `<UNK>`)
  - Estimate counts and probabilities for sequences involving `<UNK>` like for regular word
- Subword tokenization (e.g., with Byte-Pair Encoding (BPE) – Week 02)
  - Split texts into tokens smaller than words
  - Tokens are more likely to be frequent
- **Smoothing**

Break





What is a contronym?

Single words that have two contradictory meanings (they are their own opposites) are known as contronyms, and they are quite rare. Here are ten of them:

1. **apology:** a statement of contrition for an action, or a defence of one
2. **bolt:** to secure, or to flee
3. **bound:** heading to a destination, or restrained from movement
4. **cleave:** to adhere, or to separate
5. **dust:** to add fine particles, or to remove them
6. **fast:** quick, or stuck or made stable
7. **left:** remained, or departed
8. **peer:** a person of the nobility, or an equal
9. **sanction:** to approve, or to boycott
10. **weather:** to withstand, or to wear away

Mysophobia is the fear of germs (aka germophobia or bacterophobia).

Outline

- Language Models
  - Motivation
  - Sentence Probabilities
  - Markov Assumption
  - Challenges
- **Smoothing**
  - **Laplace Smoothing**
  - Backoff & Interpolation
  - Kneser-Ney Smoothing
- Evaluating Language Models

Smoothing

- Basic idea
  - Avoid assigning probabilities of 0 to unseen n-grams
  - "Move" some probability mass from more frequent n-grams to unseen n-grams
  - Also called: **discounting**

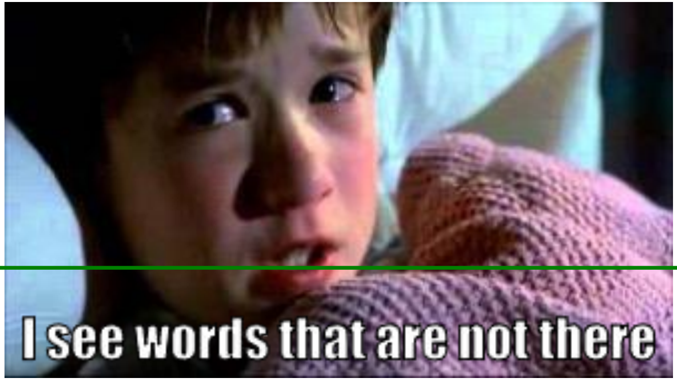


Photo Credits: Capture from *The Sixth Sense*, distributed by Buena Vista Pictures.

hallucinate

- Basic method: **Laplace Smoothing** (also: Add-1 Smoothing)
  - Example for bigrams

	i	like	the	story
i	0	693	20	0
like	326	0	1,997	8
the	15	42	0	5,171
story	23	16	16	0

Add 1 →

	i	like	the	story
i	1	694	21	1
like	327	1	1,998	9
the	16	43	1	5,172
story	24	17	17	1

Smoothing — Laplace Smoothing

- Calculating the probabilities

$$P_{Laplace}(w_n|w_{1:n-1}) = \frac{Count_{Laplace}(w_{1:n-1}w_n)}{\sum_w Count_{Laplace}(w_{1:n-1}w)}$$

$$= \frac{Count(w_{1:n-1}w_n) + 1}{\sum_w [Count(w_{1:n-1}w) + 1]}$$

$$= \frac{Count(w_{1:n-1}w_n) + \boxed{1}}{\boxed{Count(w_{1:n-1}) + V}}$$

e.g., for bigrams: $\boxed{P_{Laplace}(w_n|w_{n-1}) = \frac{Count(w_{n-1}w_n) + 1}{Count(w_{n-1}) + V}}$

Smoothing — Laplace Smoothing

- Effects of smoothing on probabilities

Bigram probabilities (<u>without</u> Laplace Smoothing):

	i	like	the	story
i	0.0	0.007949	0.000229	0.0
like	0.016413	0	0.100544	0.000403
the	0.000045	0.000127	0.0	0.015629
story	0.002073	0.001442	0.001442	0.0

Bigram probabilities (<u>with</u> Laplace Smoothing):

	i	like	the	story
i	0.000006	0.004075	0.000123	0.000006
like	0.003175	0.000010	0.019401	0.000087
the	0.000039	0.000104	0.000002	0.012493
story	0.000255	0.000180	0.000180	0.000011

- Observations
  - No zero probabilities (duh!)
  - Some non-zero probabilities have changed quite a bit!

➜ For some n-grams: (arguably) too much probability gets moved to zero probabilities

Smoothing — Laplace Smoothing

- Effects of smoothing on counts
  - Question: What counts — without smoothing — would yield $P_{Laplace}(w_i|w_{i-1})$?

$$P_{Laplace}(w_n|w_{n-1}) = \frac{Count(w_{n-1}w_n) + 1}{Count(w_{n-1}) + V} = \frac{Count^*(w_{n-1}w_n)}{Count(w_{n-1})}$$

$$\rightarrow \; Count^*(w_{n-1}w_n) = (Count(w_{n-1}w_n) + 1) \cdot \frac{Count(w_{n-1})}{Count(w_{n-1}) \boxed{+V}}$$

Bigram counts (original):

	i	like	the	story
i	0	693	20	0
like	326	0	1,997	8
the	15	42	0	5,171
story	23	16	16	0

Bigram counts (adjusted):

	i	like	the	story
i	0.51	355.28	10.75	0.51
like	63.07	0.19	385.34	1.74
the	12.79	34.37	0.80	4133.5
story	2.83	2.00	2.00	0.12

Smoothing — Laplace Smoothing

- Laplace Discount
  - d_c — ratio of adjusted counts to the original counts
  - Only defined where original counts > 1

$$d_c = \frac{Count^*(w_{n-1}w_n)}{Count(w_{n-1}w_n)}$$

Laplace discounts:

	i	like	the	story
i		0.51	0.54	
like	0.19		0.19	0.22
the	0.85	0.82		0.80
story	0.12	0.13	0.13	

Add-*k* Smoothing

- Generalize Laplace (Add-1) Smoothing
  - Add k instead of 1
  - Set $0 < k \leq 1$

$$P_{add\text{-}k}(w_n|w_{n-1}) = \frac{Count(w_{n-1}w_n) + k}{Count(w_{n-1}) + kV}$$

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Backoff & Interpolation

- Intuition: Utilize less context if required
  - Assume we want to calculate $P(w_n|w_{n-2}, w_{n-1})$ but trigram $w_{n-2}w_{n-1}w_n$ is not in the dataset

(1) **Backoff**

- Make use if bigram probability $P(w_n|w_{n-1})$
- If still insufficient, use unigram probability $P(w_n)$

(2) **Interpolation**

- Estimate $P(w_n|w_{n-2}, w_{n-1})$ as a weighted mix of trigram, bigram, and unigram probabilities
- Learn weights λ_i from data
- In practice, better than Backoff

Linear Interpolation (example for trigrams)

- Simple interpolation

$$\hat{P}(w_n|w_{n-2}, w_{n-1}) = \lambda_1 P(w_n) + \lambda_2 P(w_n|w_{n-1}) + \lambda_3 P(w_n|w_{n-2}, w_{n-1})$$

with $\sum_i \lambda_i = 1$

- λ_i conditioned on context

$$\hat{P}(w_n|w_{n-2}, w_{n-1}) = \lambda_1(w_{n-2}, w_{n-1}) P(w_n) + \lambda_2(w_{n-2}, w_{n-1}) P(w_n|w_{n-1}) + \lambda_3(w_{n-2}, w_{n-1}) P(w_n|w_{n-2}, w_{n-1})$$

Backoff & Interpolation

- Learn weights λ_i from data — basic idea

  (1) Collect held-out corpus
  - Additional corpus *or*
  - Split from initial corpus

  (2) Calculate all n-gram probabilities
  - Calculation must not consider any held-out corpus!

  (3) Find λ_i that maximizes $\hat{P}(w_n|w_{n-2}, w_{n-1})$ over held-out corpus
  - e.g., using Expectation-Maximization (EM) algorithm (not further discussed here)

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Kneser–Ney Smoothing

- Idea of Kneser–Ney Smoothing: **Absolute Discounting Interpolation**

Remove a fixed value from all bigram counts

Interpolation but with better estimates for unigram probabilities

$$P_{KN}(w_n|w_{n-1}) = \frac{max[Count(w_{n-1}w_n) - d, 0]}{Count(w_{n-1})} + \lambda(w_{n-1})P_{KN}(w_n)$$

Note: We only look at a bigram language model in the following to keep the examples and notations easy. Kneser-Ney Smoothing is analogously defined for larger n-grams.

Kneser–Ney Smoothing — Absolute Discounting

- Absolute discounting
  - Remove fixed value $\boxed{d}$ from bigram counts
    (typically: $0 < d < 1$)
  - Makes probability mass for unigrams available
  - Intuition

    If $Count(w_{n-1}w_n)$ is large, count is hardly affected

    If $Count(w_{n-1}w_n)$ is small, count is not that useful to begin with

just a fail-safe to avoid negative probabilities

$$\boxed{\frac{max\boxed{[Count(w_{n-1}w_n) - d},0]}{Count(w_{n-1})}}$$

➜ Question: How to pick the value(s) for d ?

Kneser–Ney Smoothing — Absolute Discounting

- Approach by Church and Gale (1991)
  - Compute bigram counts over large training corpus
  - Compute the counts of the same bigrams over a large test corpus
  - Compute the average count from the test corpus with respect to the count in the training corpus

Bigram count in training corpus	Bigram count in test corpus
0	0.000270
1	0.448
2	1.25
3	2.24
4	3.23
5	4.21
6	5.23
7	6.21
8	7.21
9	8.26

On average, a bigram that occurred 5 times in the training corpus occurred 4.21 times in the test corpus

➜ Set $d \boxed{= 0.75}$ (maybe a bit smaller for counts of 1 and 2)

Source: A comparison of the enhanced Good-Turing and deleted estimation methods for estimating probabilities of English bigrams (Church and Gale, 1991)

Kneser-Ney Smoothing — Interpolation with a Twist

- Motivation

$$P_{KN}(w_n|w_{n-1}) = \frac{max\,[Count(w_{n-1}w_n) - d, 0]}{Count(w_{n-1})} + \lambda(w_{n-1})P(w_n)$$

Using basic interpolation, that would just be the unigram probability

➜ **But is this actually a good idea?**

Predict the missing word:

"I can't see without my reading ________________"

…

"glasses"

…

…

"Kong"

…

If *"Hong Kong"* is very frequent:

$P(\text{“}Kong\text{”}) > P(\text{“}glasses\text{”})$

Kneser-Ney Smoothing — Interpolation with a Twist

- The difference between *"glasses"* and *"Kong"* — Intuition
  - *"glasses"* is preceded by many other words
  - *"Kong"* almost only preceded by *"Hong"*

➜ $P(w) =$ *"How likely is w ?"* … Maybe not most intuitive approach

- Alternative: $\boxed{P_{KN}(w)} =$ *"How likely is w to appear as a novel continuation?"*
  - $P_{KN}(w)$ is high ⇔ there are <u>many words</u> w' that form an existing bigram $w'w$
  - $P_{KN}(w)$ is low ⇔ there are <u>only few words</u> w' that form an existing bigram $w'w$

➜ How can we quantify this?

Kneser-Ney Smoothing — Interpolation with a Twist

- Calculating $P_{KN}(w)$

words w' that form an existing bigram $w'w$

$$\boxed{P_{KN}(w)} = \frac{\boxed{|\{w' : Count(w'w) > 0\}|}}{\boxed{|\{(u, v) : Count(uv) > 0\}|}}$$

total number of existing bigrams

normalization to ensure that $\sum_{n=1}^{N} P(w_n) = 1$

🏃‍♀️🏃🏃‍♂️ In-Lecture Activity (5 mins)

- Task: find 5+ words where you would expect that $P_{KN}(w) > P(w)$
  - Post your answer to Canvas > Discussions > [In-Lecture] L1 … (2 Feb)
    (one student of your group can post the reply, but include your group members' names)
  - We already used "Kong" as an example, so try to avoid "Francisco", "Angeles", "Aires", etc. :)
  - Optional: Think about how the context matters (e.g., travel blogs vs. movie reviews)

😎 **Pro Tip**: It's not a competition,
but about discussions and sharing ideas

Kneser-Ney Smoothing — Wrapping it Up

$$P_{KN}(w_n|w_{n-1}) = \frac{max\left[Count(w_{n-1}w_n) - d, 0\right]}{Count(w_{n-1})} + \underbrace{\lambda(w_{n-1})}_{\text{last missing puzzle piece}} P_{KN}(w_n)$$

- Normalizing factor λ
  - Required to account for the probability mass we have discounted

$$\lambda(w_{n-1}) = \underbrace{\frac{d}{Count(w_{n-1})}}_{\text{normalized discount}} \cdot \underbrace{\left|\{w' : Count(w_{n-1}w') > 0\}\right|}_{\text{\# words that can follow}}$$

= # words that have been discounted

= # times the normalized discount has been applied

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- **Evaluating Language Models**

Evaluating Language Models

- A Language Model (LM) is considered good if
  - It assigns high probabilities to frequently occurring sentences
  - It assigns low probabilities to rarely occurring sentences

- 2 basic approaches to compare LMs

Extrinsic Evaluation

- Requires a downstream task (e.g., spell checker, speech recognition)
- Run downstream task with each LM and compare the results
- Can be very expensive & time-consuming

Intrinsic Evaluation

- Evaluate each LM on a test corpus
- Generally cheaper & faster
- Require intrinsic metric to compare LMs

➜ **Perplexity** (among other metrics)

Intrinsic Evaluation

- 3 core steps for an intrinsic evaluation

    (1) Train LM on a **training corpus**
    (i.e., compute the n-gram probabilities)

    (2) Tune parameters of LM using a **development corpus**
    (e.g., *k* in case of Add-*k* Smoothing)

    (3) Compute evaluation metric on **test corpus**
    (e.g., perplexity)

- Common corpus breakdown: 80/10/10 (80% training, 10% development, 10% test)

Perplexity — Intuition

- How easy is it to predict the next word?

I always order pizza with cheese and …

The 33^{rd} President of the US was …

I saw a …

mushrooms 0.1

pepperoni 0.1

anchovies 0.01

….

fried rice 0.0001

….

and 1e-100

- Unigrams are terrible at this game. Why?

Perplexity

- Perplexity — Definition
  - The best language model is the one that best predicts an unseen test set: highest $P\,(sentence)$
  - Inverse probability of test corpus W
  - Normalized by the number of words N in test corpus

$$PP(W) = \boxed{P(w_1, w_2, \ldots, w_N)}^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1, w_2, \ldots, w_N)}}$$

chain rule: $$= \sqrt[N]{\prod_{n=1}^{N} \frac{1}{P(w_n|w_1, \ldots, w_{n-1})}}$$

e.g., for bigrams: $$\overset{\sim}{=} \sqrt[N]{\prod_{n=1}^{N} \frac{1}{P(w_n|w_{n-1})}}$$

Markov

Minimizing perplexity ⇔ Maximizing probability

Perplexity — Intuition

- When is the perplexity **high** 😖?

1. Many n-grams are <u>frequent</u> in the training corpus but <u>rare</u> in the test corpus

↓

Very few high $P(w_n|w_{n-1})$ values over test corpus

2. Many n-grams are <u>rare</u> in the training corpus but <u>frequent</u> in the test corpus

↓

Many low $P(w_n|w_{n-1})$ values over test corpus

High perplexity $PP(W) = \sqrt[N]{\prod_{n=1}^{N} \frac{1}{P(w_n|w_{n-1})}}$

Perplexity — Practical Consideration

- In general
  - Each $P(w_n|w_{1:n-1})$ rather small ➜ $\prod_{n=1}^{N} P(w_n|w_{1:n-1})$ very small
  - Risk of arithmetic underflow

- Again, logarithm to the rescue

$$PP(W) = e^{\ln PP(W)}$$

$$\ln PP(W) = -\frac{1}{N} P(w_1, w_2, \ldots, w_N)$$

$$= -\frac{1}{N} \ln \prod_{n=1}^{N} P(w_n|w_1, \ldots, w_{n-1})$$

$$= -\frac{1}{N} \sum_{n=1}^{N} \ln P(w_n|w_1, \ldots, w_{n-1})$$

e.g., for bigrams: $$= -\frac{1}{N} \sum_{n=1}^{N} \ln P(w_n|w_{n-1})$$

Perplexity — Toy Example

- Evaluation setup
  - Bigram LM trained over 25k movie reviews
  - Small test corpus W with $N = 12$

$$W = [\\ \text{"}\langle s\rangle\ i\ like\ good\ movies\ \langle /s\rangle\text{"},\\ \text{"}\langle s\rangle\ the\ story\ is\ funny\ \langle /s\rangle\text{"}\\ ]$$

$$PP(W) = \sqrt[N]{\prod_{n=1}^{N} \frac{1}{P(w_n|w_{n-1})}} = \mathbf{40.1}$$

bigram	P(bigram)
"<s> i"	0.0882
"i like"	0.0079
"like good"	0.0013
"good movies"	0.0062
"movies </s>"	0.0034
"<s> the"	0.0990
"the story"	0.0156
"story is"	0.1138
"is funny"	0.0022
"funny </s>"	0.0081

Perplexity — Real-World Example

- Evaluation setup
  - Unigram, Bigram, Trigram LMs trained over *Wall Street Journal* articles
  - Training corpus: ~38 million words (~20k unique words)
  - Test corpus: ~1.5 million words

	Unigram	Bigram	Trigram
Perplexity	962	170	109

Source: https://web.stanford.edu/~jurafsky/slp3/3.pdf

🏃‍♀️ 🏃 🏃‍♂️ Quick Quiz (2 mins)

What are the (**minimum**, **maximum**) possible values for perplexity?

A (0, ∞)

B (1, ∞)

C (0, V)

D (1, V)

v = size of vocabulary

Summary

- Language Models — assigning probabilities to sentences
  - Very important concept for many NLP tasks
  - Different methods to compute sentence probabilities
    (here: n-grams; later we come back to them using neural networks)
- n-gram Language Models
  - Intuitive training ➜ Maximum Likelihood Estimations
  - Main consideration: zero probabilities due to large n-grams and/or open vocabularies

MLE = count

Markov Assumption to limited size of considered n-grams

Focus here: **Smoothing** (maybe with backoff & interpolation)

In practice, typically a combination of these and similar approaches

Outlook for Next Week: Text Classification

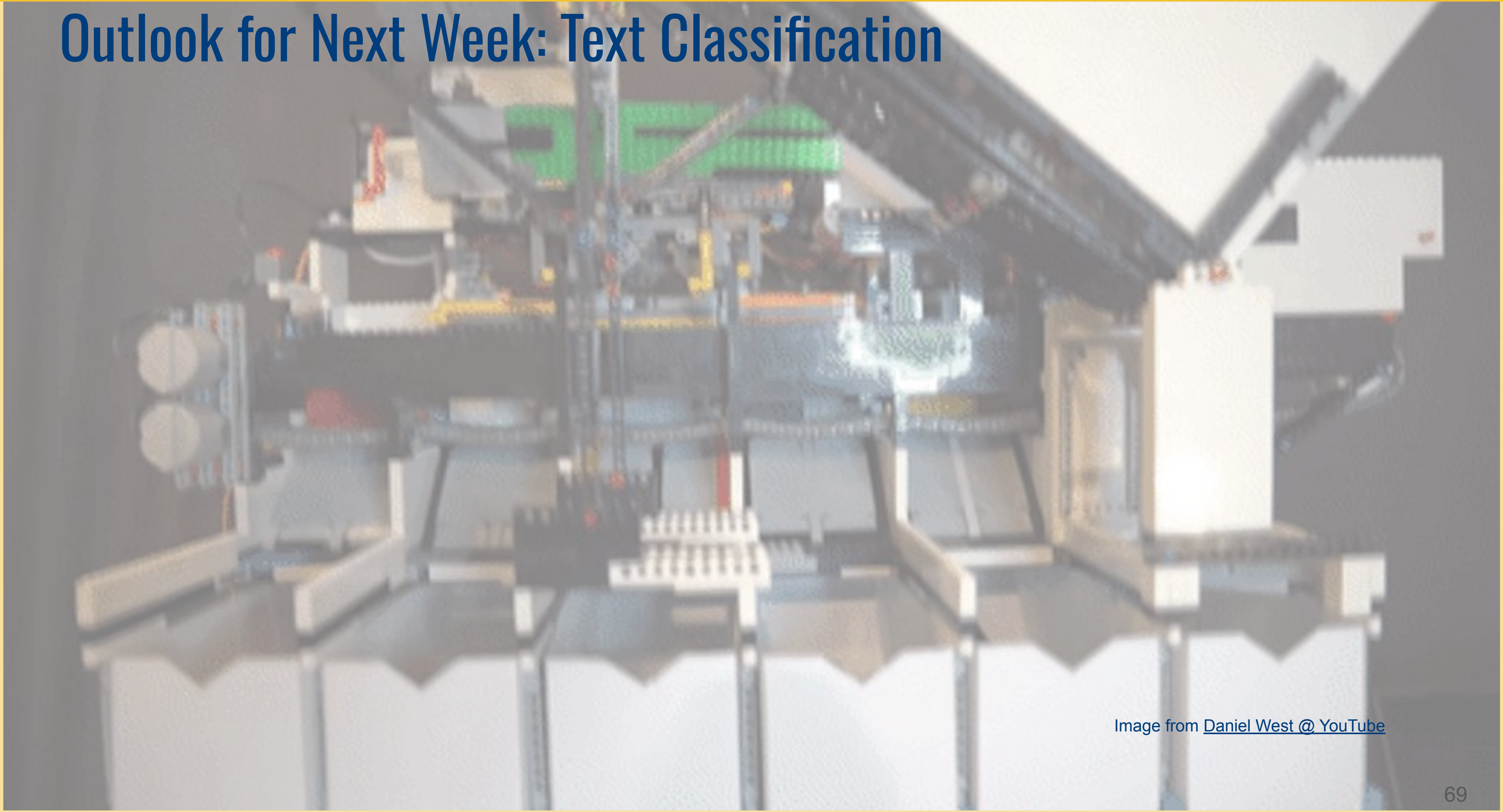

Image from [Daniel West @ YouTube]

Pre-Lecture Activity for Next Week

- Assigned Task (due before Feb 9)
  - Post a 1–2 sentence answer to the following question in the Pre-Lecture forum.
    (you will find the thread on Canvas > Discussions > [Pre-Lecture])

"When we want to evaluate classifiers,
*why is **accuracy** alone often not a good metric?"*

Side notes:

- This task is meant as a warm-up to provide some context for the next lecture
- No worries if you get lost; we will talk about this in the next lecture
- You can just copy-&-paste others' answers but this won't help you learn better